

# Optimal bidding strategy for microgrids in joint energy and ancillary service markets considering flexible ramping products <sup>☆</sup>



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## HIGHLIGHTS

- Flexible ramping products are modelled in the framework of a microgrid.
- Microgrids' optimal bidding model is proposed in energy and ancillary service markets.
- A hybrid stochastic and robust optimization approach is adopted.
- The effectiveness of the proposed bidding model is verified based on real-world data.

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## ABSTRACT

Due to the volatile nature of wind and photovoltaic power, wind farms and solar stations are generally thought of as the consumers of ramping services. However, a microgrid (MG) is able to strategically integrate various distributed energy resources (DERs) to provide both energy and ancillary services (ASs) for the bulk power system. To evaluate the ramping capabilities of an MG in the joint energy and AS markets, an optimal bidding strategy is developed in this paper considering flexible ramping products (FRPs). By aggregating and coordinating various DERs, including wind turbines (WTs), photovoltaic systems (PVs), micro-turbines (MTs) and energy storage systems (ESSs), the MG is able to optimally allocate the capacities for energy, spinning reserve and ramping. Taking advantage of the synergy among DERs, the MG can maximize its revenues from different markets. Moreover, the flexibility of the MG for the bulk power system can be fully explored. To address the uncertainties introduced by renewable generation and market prices, a hybrid stochastic/robust optimization (RO) approach is adopted. Case studies based on a real-world MG with various DERs demonstrate the market behavior of the MG using the proposed bidding model.

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## 1. Introduction

### 1.1. Motivation

The development of renewable energy has been drawing attention across the world in the past decade. California, for example, announced its ambitious goal of achieving a 50% renewable portfolio standard by 2030 [1]. While the use of renewable energy

contributes to a more sustainable future, the variabilities and uncertainties of the renewable sources pose great challenges to the economic and reliable operations of the power system [2]. With the increasing penetration of renewable energy, rapid ramping of generation resources may be insufficient to smooth out the huge fluctuations in renewable energy production. Thus, it is critical to facilitate the accommodation of renewable generation while economically and reliably operating the power system.

The concept of the microgrid (MG) assumes a cluster of loads and distributed energy resources (DERs) operating as a single controllable system [3]. Taking advantage of the synergy among various DERs, the renewable generators can cooperate with controllable energy resources to provide both energy and ancillary services (ASs) for the bulk power system [4]. For example, in a

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**Nomenclature**

*Indices and sets*

$t$	time index
$s$	price scenario index
$\Phi^B$	set of decision variables
$\Phi^U$	set of random variables
$E$	superscript for energy
$RES$	superscript for spinning reserve service
$RAMPU$	superscript for upward FRP service
$RAMPD$	superscript for downward FRP service
$WT$	superscript for wind turbines
$PV$	superscript for photovoltaic systems
$MT$	superscript for micro-turbines
$ESS$	superscript for energy storage systems

*Parameters and constants*

$\hat{P}_t^{WT}$	point forecast of historical wind power (Unit: MW) in the MG at time slot $t$
$\hat{P}_t^{PV}$	point forecast of historical solar power (Unit: MW) in the MG at time slot $t$
$\Gamma_t^{WT}$	robustness parameter of wind power
$\Gamma_t^{PV}$	robustness parameter of solar power
$P_{max}^{WT}$	total wind power capacity (Unit: MW) in the MG
$P_{max}^{PV}$	total solar power capacity (Unit: MW) in the MG
$N^{MT}$	number of MTs in the MG
$N^{ESS}$	number of ESSs in the MG
$\gamma_s$	weight of price scenario $s$
$N^S$	number of price scenarios
$\lambda_{t,s}^{(\cdot)}$	day-ahead market price (Unit: \$/MW h) at time slot $t$ in scenario $s$
$h$	time interval
$\beta^{(\cdot)}$	expectation of real-time deployment ratio of ancillary services
$c_i^{MT}$	operation cost per unit energy production (Unit: \$/MW h) of MT $i$
$P_{i,max}^{MT}$	maximal power (Unit: MW) of MT $i$

$P_{i,max}^{MT,RAMPU}$	Maximal ramping-up capacity (Unit: MW/h) of MT $i$
$P_{i,max}^{MT,RAMPD}$	maximal ramping-down capacity (Unit: MW/h) of MT $i$
$P_{i,\alpha,max}^{ESS}$	maximal charging power (Unit: MW) of ESS $i$
$P_{i,\beta,max}^{ESS}$	maximal discharging power (Unit: MW) of ESS $i$
$\eta_{i,\alpha}$	charging efficiency of ESS $i$
$\eta_{i,\beta}^{ESS}$	discharging efficiency of ESS $i$
$SOC_{i,min}^{ESS}$	minimal state of charge of ESS $i$
$SOC_{i,max}^{ESS}$	maximal state of charge of ESS $i$
$C_i^{ESS}$	capacity (Unit: MW h) of ESS $i$
$E_{i,0}^{ESS}$	initial stored energy (Unit: MW h) of ESS $i$ in scenario $s$
$P_t^D$	load demand (Unit: MW) of the MG at time slot $t$

*Variable*

$P_t^{AWT}$	random variable of available wind power (Unit: MW) at time slot $t$
$P_t^{APV}$	random variable of available solar power (Unit: MW) at time slot $t$
$\varepsilon_t^{WT}$	normalized error between actual and point forecast wind power
$\varepsilon_t^{PV}$	normalized error between actual and point forecast solar power
$R_s^{(\cdot)}$	revenue (Unit: \$) in scenario $s$ from the day-ahead markets
$C^{OP}$	operation costs (Unit: \$) of the MG
$P_t^{(\cdot)}$	bidding capacity (Unit: MW) of the MG or DER for energy or ASs at time slot $t$
$\alpha_{i,t}^{ESS}$	binary variable of ESS $i$ at time slot $t$ representing the status of charging
$\beta_{i,t}^{ESS}$	binary variable of ESS $i$ at time slot $t$ representing the status of discharging
$E_{i,t}^{ESS}$	stored energy (Unit: MW h) of ESS $i$ at time slot $t$

stand-alone mode, a wind farm must deviate from its maximum power output status and leave a margin to provide ramping services for the system. However, in an MG, the wind farm is able to leave the ramping margin by charging a Na/S battery without deviating from its maximum power. Hence, an MG can stably provide both energy and ASs by integrating various DERs [5]. From the system point of view, MGs show the advantages of low investment costs, low pollutant emission and high operational flexibility. The flexibility of the MGs provided by the DERs can be aggregated for power system operations, thereby replacing high-cost centralized units and deferring the generation expansion. In addition, the MGs are located at the demand side, efficiently offering capacities to meet the local requirements [6]. Compared with centralized thermal units, MGs can achieve localized energy balance without the loss accompanied with long-distance power transmission and difficulties caused by transmission congestions. Therefore, the concept of the MG provides new insights for exploring the grid-friendly manner of DERs [7].

In most electricity markets across the world, ASs play a critical role in the reliable operation of power systems. In California, for example, the reserve and regulation services are co-optimized with the energy in the day-ahead market. With the increasing penetration of solar energy, the variability and uncertainties in net load demands will become more severe in the real-time operation. As illustrated in Fig. 1, the steep rise of the system net loads from 17:00 to 18:00 as the sun sets requires over 5500 MW of

generating capacity to come online, which poses great challenges to the secure operation of the power system.

To cope with the inadequacy of the system's ramping capacities, the flexible ramping product (FRP) has been introduced into the California market recently to improve the dispatch flexibility and address the operational challenges [8,9]. FRPs are flexible generation capacities dispatched by the independent system operator (ISO) to deal with energy imbalances and satisfy the load following requirements in the real-time operation. The energy imbalances

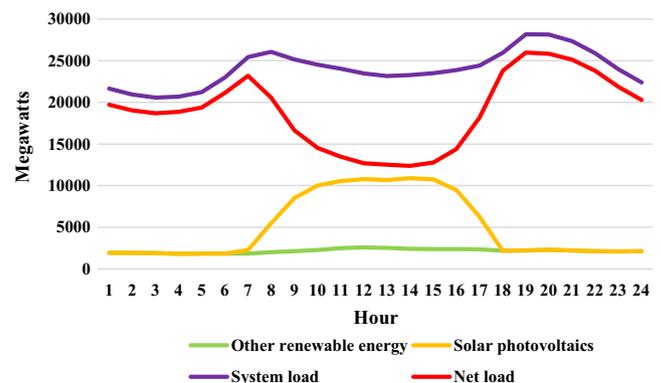


Fig. 1. Hourly renewable energy and electric load demands on March 1, 2017 in California.

can arise due to load and supply variability. The FRPs consist of separate products in the upward and downward directions as the energy imbalances may be positive or negative [8]. Currently, fast-ramp generators are generally dispatched to provide FRPs.

With the integration of rapid ramping resources, e.g., micro-turbines (MTs) and energy storage systems (ESSs), the MGs are promising energy resources to provide fast ramping capacities for the power system. In existing studies, the bidding strategies of an MG with various DERs participating in the energy and reserve markets have been investigated [10–12]. These studies examined the abilities of an MG to provide energy and spinning reserve. However, few studies have focused on an MG's participation in different types of ASs, especially the FRPs. Therefore, it is imperative to evaluate the ramping capabilities of MGs in the market environment. This paper is the pioneering work on the optimal bidding strategy for MGs participating in joint energy, spinning reserve and FRP markets.

## 1.2. Literature review and contribution

The optimal bidding strategies for MGs in the energy market have been widely investigated in the last decade. In [10], an optimal day-ahead price-based power scheduling problem is studied for a community-scale MG. The model aims at maximizing the expected benefits of the MG in the energy market while satisfying users' thermal comfort requirements. In [13], the day-ahead bidding strategy of a commercial virtual power plant (VPP) is addressed considering various DERs. To address the uncertainties in load consumptions and real-time prices, a three-stage stochastic optimization model is formulated for optimal energy scheduling. In [14], the optimal bidding strategy in the day-ahead energy market of an MG is proposed. The MG coordinates the energy consumption and production of its components and trades electricity in day-ahead and real-time markets. A hybrid stochastic/robust optimization method is adopted to address the uncertainties in renewable energy outputs and future prices. In [15], the concept of MG aggregators is introduced to involve small-scale MGs in the real-time balancing market bidding via a hierarchical market framework. At the upper level, the bidding strategy of the aggregator is optimized while the market is cleared at the lower level. In [16], the bidding problems of VPPs are investigated considering renewable distributed generators and inelastic demands. A stochastic bi-level optimization model is formulated to minimize the cost of the VPP in day-ahead and balancing markets.

An MG, as a controllable system, is able to provide both energy and ASs for the power system by strategically coordinating various DERs [17]. There have been extensive studies focused on the bidding strategies for MGs in joint energy and reserve markets [18]. In [19], an arbitrage strategy for VPPs by participating in energy, spinning reserve and reactive power markets is presented. The security-constrained unit commitment (SCUC) model is established to maximize VPP's profits. In [20], the bidding problem by a VPP in a joint market of energy and spinning reserve service is addressed. The proposed bidding strategy is a non-equilibrium model based on the deterministic price-based unit commitment. In [21], a risk-averse optimal offering model for a VPP is proposed in the joint energy and reserve markets. Uncertainties in renewable generation and prices from day-ahead and balancing markets are considered. Reference [21] assesses how total and surplus profits of a VPP are affected by risk-aversion. In [12], a multi-objective joint energy and reserve market clearing model is presented in which the payment cost minimization and voltage stability maximization are considered. In [22], a bidding behavior modeling and an auction architecture of the MG are proposed. The bidding behavioral states of the MG are formalized as Markov processes. In [23], an optimal bidding model of a residential MG is formulated

for day-ahead markets, considering the uncertainties in renewable power production. The renewable uncertainties are modeled based on an Analog Ensemble method, which can estimate the probability of solar power by sampling. In [24], a day-ahead optimal energy management strategy of industrial MGs is presented with high-penetration renewables. Besides satisfying its local demands, the MG participates in energy trading with the power system.

To the best of our knowledge, few existing studies have focused on the combination of different types of ASs, especially the ramping services of an MG. By means of the synergy among various DERs, MGs can be thought of as promising energy resources to cope with the inadequacy of the system's ramping capacities. Therefore, it is imperative to investigate the ramping capabilities of MGs in the market environment. In this paper, an optimal bidding model is established to evaluate the ramping capabilities of an MG in the joint energy and AS markets. By coordinating various DERs, including WTs, PVs, MTs and ESSs, an MG is able to maximize its revenues from the day-ahead markets. A hybrid stochastic/robust optimization method is adopted to address the uncertainties in renewable energy generation and market prices.

The major contributions of this paper are as follows:

- (1) FRPs are incorporated into the bidding model of an MG for the first time. The ramping capabilities of the MG can then be evaluated by optimizing the bidding model. By integrating various DERs, the ramping capabilities of the MG can be greatly improved.
- (2) The flexibility of an MG for the bulk power system is examined considering its participation in the joint energy, spinning reserve and FRP markets.
- (3) A hybrid stochastic/robust optimization approach is adopted to address the uncertainties in renewable energy and market prices. The bidding problem with uncertain coefficients can be transformed into a mixed-integer linear programming (MILP) model that can be readily solved.

## 2. Problem description

### 2.1. Co-optimization of energy and ancillary service markets

Without loss of generality, the co-optimization of energy and AS markets is implemented in this paper [25]. In the pool-based day-ahead markets, it is assumed that an MG can simultaneously bid in joint energy and AS markets. Considering its relatively small capacity, an MG is reasonably assumed to be a price-taker. As a controllable entity, the MG will strategically allocate available capacities in day-ahead markets to maximize the revenues. The MG bids energy and ASs, including spinning reserve service and FRPs [8].

The FRP provided by an MG is referred to the potential power output change from time slot  $t$  to  $t + 1$ , which reflects the available capacity of the MG reserved for the power system to satisfy load following. FRPs are specifically designed to relieve the system-wide ramping constraints, which are first introduced in California and MISO markets in the United States. The models and applications of FRPs have been investigated recently. In [26], the mathematical model of FRPs is formulated according to the California market. In [27], the security-constrained economic dispatch (SCED) model is presented to incorporate the ramping constraints. Numerical results demonstrate the effectiveness of the ramping constraints for reducing the instances of short-term scarcity conditions. In [28], a risk-constrained SCED scheme is proposed to optimize the dispatch and provision of FRPs. With the increasing demand for the ramping resources in the power system, the FRP market is expected to be fully operational in the near future.

The framework of an MG's participation in day-ahead energy and AS markets are shown in Fig. 2. In this framework, integrated

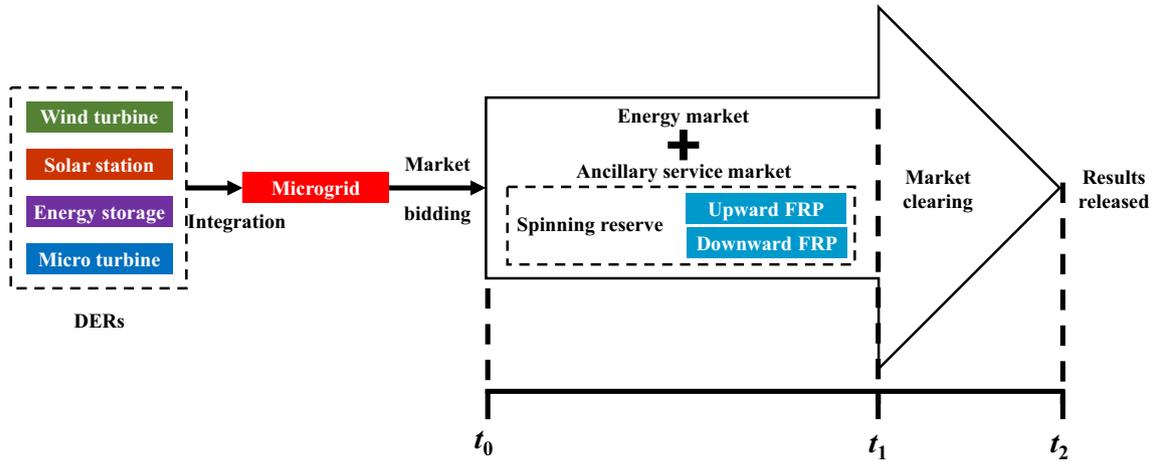


Fig. 2. Framework of an MG's participation in day-ahead energy and AS markets.

with various DERs, an MG bids in the joint energy, reserve and FRP markets at time  $t_0$ , when the ISO opens the day-ahead market. Then the day-ahead market bid period closes at time  $t_1$ . The ISO begins to run the market clearing software to determine the hourly dispatch schedules and the locational marginal prices for the day-ahead market. Finally, the market clearing results will be released at time  $t_2$ .

The co-optimization of the energy and AS markets is implemented in most electricity markets operated by the ISO [29]. The bids of the MG must be determined before the closure of the day-ahead markets for the next day [14]. By introducing FRP markets, on one hand, the MG is able to increase the revenues from market bidding; on the other hand, the power system will benefit from the improvement of the dispatch flexibility and the adequacy of the ramping resources.

### 2.2. Uncertainty modeling

In this paper, a hybrid stochastic/robust optimization approach is adopted to address the uncertainties in renewable generation and day-ahead market prices [14,25]. The prices in the energy and AS markets are modeled via scenario-based stochastic programming. The uncertainties in wind and photovoltaic power are addressed using RO.

For market prices, an MG is concerned with the profiles of market prices to optimally allocate its available capacities in each market. The prices in different markets have strong correlations, which cannot be modeled with independent confidence intervals. For example, both prices of energy and spinning reserve are relatively high during peak hours because of high load demands. In addition, to allocate the capacities in different markets, the relative differences of prices are the major concern instead of the absolute values of market prices. Therefore, stochastic programming with multiple price scenarios is more appropriate than RO to model the uncertainty of market prices [14].

For renewable generations, the absolute capacities of wind and photovoltaic power have large impacts on the bidding strategy of an MG as well as the operation of the DERs in the MG. Moreover, because the intervals of wind and photovoltaic power can be obtained according to historical data, RO is an effective tool to address the uncertainties in renewable generation. Therefore, the robust mixed-integer linear programming (RMILP) in [30] is applied in this paper.

The available power of the WT and PV in the MG at time slot  $t$ , denoted by  $P_t^{AWT}$  and  $P_t^{APV}$ , are modeled as independent and bounded random variables. Under a confidence level  $\sigma$ ,  $P_t^{AWT}$  takes

values from the minimum power  $\underline{P}_t^{AWT}$  to the maximum power  $\bar{P}_t^{AWT}$ , while  $P_t^{APV}$  takes values from  $\underline{P}_t^{APV}$  to  $\bar{P}_t^{APV}$ . To obtain the confidence intervals of  $P_t^{AWT}$  and  $P_t^{APV}$ , the forecast errors are analyzed based on historical datasets.

For example,  $\hat{P}_t^{WT}$  and  $P_t^{WT}$  are the point forecast and actual wind power at time slot  $t$ . Based on the historical data from the Wind Integration Datasets of the NREL [31], the probability distribution  $f^{WT}(\hat{P}_t^{WT}/P_{max}^{WT})$  of wind power forecast errors  $\varepsilon_t^{WT}$  can be acquired, i.e.,

$$\varepsilon_t^{WT} = \frac{\hat{P}_t^{WT} - P_t^{WT}}{P_{max}^{WT}}, \varepsilon_t^{WT} \sim f^{WT}(\hat{P}_t^{WT}/P_{max}^{WT}). \quad (1)$$

Then the upper and lower bounds of wind power forecast errors under the confidence level  $\sigma$ ,  $\varepsilon_{t,min}^{WT}$  and  $\varepsilon_{t,max}^{WT}$ , can be calculated as follows:

$$\varepsilon_{t,min}^{WT} = \text{Inf} \left\{ w \in [0, 1] \mid \int_0^w f^{WT}(x) dx \geq \frac{1-\sigma}{2} \right\}, \quad (2)$$

$$\varepsilon_{t,max}^{WT} = \text{Inf} \left\{ w \in [0, 1] \mid \int_0^w f^{WT}(x) dx \geq \frac{1+\sigma}{2} \right\}. \quad (3)$$

Fig. 3 shows the intervals of forecast errors with 95% confidence level under different levels of forecasted wind power [32].

According to the forecasted wind power, the minimum power and the maximum power can be calculated as follows:

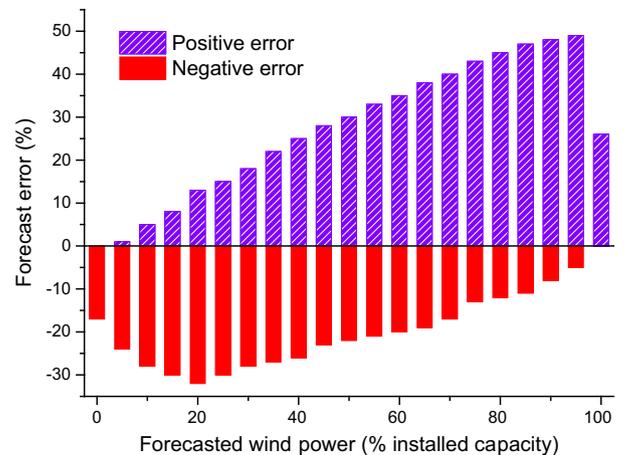


Fig. 3. The intervals of forecast errors with 95% confidence level under different levels of forecasted wind power.

$$\underline{P}_t^{AWT} = \hat{P}_t^{AWT} - P_{\max}^{WT} \cdot \varepsilon_{t,\max}^{WT}, \varepsilon_{t,\max}^{WT} > 0, \quad (4)$$

$$\bar{P}_t^{AWT} = \hat{P}_t^{AWT} - P_{\max}^{WT} \cdot \varepsilon_{t,\min}^{WT}, \varepsilon_{t,\min}^{WT} < 0. \quad (5)$$

Then the random variable for the available wind power at time slot  $t$  is bounded as follows:

$$P_t^{AWT} \in [\underline{P}_t^{AWT}, \bar{P}_t^{AWT}]. \quad (6)$$

The intervals for the available PV power can be obtained in a similar manner with wind power.

### 3. Optimal bidding model for microgrids

By coordinating various DERs, an MG is able to provide flexibility for the bulk power system. In this paper, FRPs are modeled in the bidding strategy of the MG in the day-ahead markets. As a price-taker, the MG strategically allocates the hourly capacities in energy, spinning reserve and FRPs.

#### 3.1. Objective

The objective of the optimal bidding strategy for an MG is to maximize the total revenue from the energy, spinning reserve and FRP markets, shown as follows:

$$[SPS_DFTerm_3][SPS_DFTerm_2][SPS_DFTerm_4] \max_{\phi^b} \min_{\phi^u} \sum_{s=1}^{N^s} \gamma_s (R_s^E + R_s^{RES} + R_s^{FRP} - C^{OP}), \quad (7)$$

where

$$[SPS_DFTerm_4][SPS_DFTerm_3][SPS_DFTerm_7][SPS_DFTerm_2] \times [SPS_DFTerm_2][SPS_DFTerm_2][SPS_DFTerm_1][SPS_DFTerm_1][SPS_DFTerm_6] \times [SPS_DFTerm_5] R_s^E = \sum_{t=1}^T \lambda_{t,s}^E [P_t^E + \beta^{RES} P_t^{RES} + \beta^{RAMP} (P_t^{RAMP} - P_t^{RAMPD})] h, \quad (8)$$

$$R_s^{RES} = \sum_{t=1}^T \lambda_{t,s}^{RES} P_t^{RES} h, \quad (9)$$

$$R_s^{FRP} = \sum_{t=1}^T (\lambda_{t,s}^{RAMP} P_t^{RAMP} + \lambda_{t,s}^{RAMPD} P_t^{RAMPD}) h, \quad (10)$$

$$[SPS_DFTerm_4][SPS_DFTerm_2][SPS_DFTerm_9] C^{OP} = \sum_{t=1}^T \sum_{i=1}^{N^{MT}} C_i^{MT} [P_{i,t}^{MT,E} + \beta_i^{MT,RES} P_{i,t}^{MT,RES} + \beta_i^{MT,RAMP} (P_{i,t}^{MT,RAMP} - P_{i,t}^{MT,RAMPD})] h. \quad (11)$$

The revenues from the energy and reserve markets are shown in (8) and (9). The revenues from the FRPs are shown in (10), composed of the upward and downward FRPs. (11) shows that the operational costs of the MG come from the fuel costs of the MTs. Note that in (8) and (11), to consider the influences of the real-time deployment of ASs, the ratios  $\beta^{(\cdot)}$  are used to estimate the potential energy requirement in providing AS [25].

#### 3.2. Constraints

##### 3.2.1. The constraints of renewable generators

$$P_t^{WT,E} + P_t^{WT,RES} + P_t^{WT,RAMP} \leq P_t^{AWT}, \forall t, \quad (12)$$

$$P_t^{PV,E} + P_t^{PV,RES} + P_t^{PV,RAMP} \leq P_t^{APV}, \forall t, \quad (13)$$

$$P_t^{WT,E} - P_t^{WT,RAMPD} \geq 0, \forall t, \quad (14)$$

$$P_t^{PV,E} - P_t^{PV,RAMPD} \geq 0, \forall t. \quad (15)$$

In (12) and (13), the energy, reserve and upward FRP of the WT are restricted by the available wind and photovoltaic power. In (14) and (15), the difference between the energy and the downward FRP should be no less than 0.

##### 3.2.2. The constraints of micro turbines

$$P_{i,t}^{MT,E} + P_{i,t}^{MT,RES} + P_{i,t}^{MT,RAMP} \leq P_{i,\max}^{MT}, \forall i, t, \quad (16)$$

$$P_{i,t}^{MT,E} - P_{i,t}^{MT,RAMPD} \geq 0, \forall i, t, \quad (17)$$

$$(P_{i,t+1}^{MT,E} + P_{i,t+1}^{MT,RES} + P_{i,t+1}^{MT,RAMP}) - (P_{i,t}^{MT,E} - P_{i,t}^{MT,RAMPD}) \leq P_{i,\max}^{MT,RAMP}, \forall i, t, \quad (18)$$

$$(P_{i,t}^{MT,E} + P_{i,t}^{MT,RES} + P_{i,t}^{MT,RAMP}) - (P_{i,t+1}^{MT,E} - P_{i,t+1}^{MT,RAMPD}) \leq P_{i,\max}^{MT,RAMPD}, \forall i, t. \quad (19)$$

In (16) and (17), the bidding capacities of the MTs should be limited by the maximum and minimum power. In (18) and (19), the ramping up/down constraints are modeled.

##### 3.2.3. The constraints of energy storage systems

$$[SPS_DFTerm_2][SPS_DFTerm_6][SPS_DFTerm_1] 0 \leq \alpha_{i,t}^{ESS} + \beta_{i,t}^{ESS} \leq 1, \alpha_{i,t}^{ESS}, \beta_{i,t}^{ESS} \in \{0, 1\}, \forall i, t, \quad (20)$$

$$0 \leq P_{i,t,\alpha}^{ESS} \leq \alpha_{i,t}^{ESS} P_{i,\alpha,\max}^{ESS}, \forall i, t, \quad (21)$$

$$0 \leq P_{i,t,\beta}^{ESS} \leq \beta_{i,t}^{ESS} P_{i,\beta,\max}^{ESS}, \forall i, t, \quad (22)$$

$$P_{i,t}^{ESS,E} = P_{i,\beta,t}^{ESS} - P_{i,\alpha,t}^{ESS}, \forall i, t, \quad (23)$$

$$P_{i,t}^{ESS,E} + P_{i,t}^{ESS,RES} + P_{i,t}^{ESS,RAMP} \leq P_{i,\beta,\max}^{ESS}, \forall i, t, \quad (24)$$

$$P_{i,t}^{ESS,E} - P_{i,t}^{ESS,RAMPD} \geq -P_{i,\alpha,\max}^{ESS}, \forall i, t, \quad (25)$$

$$P_{i,t}^{ESS,E} h^E + P_{i,t}^{ESS,RES} h^{RES} + P_{i,t}^{ESS,RAMP} h^{RAMP} \leq E_{i,t}^{ESS}, \forall i, t, \quad (26)$$

$$E_{i,t}^{ESS} - P_{i,t}^{ESS,E} h^E + P_{i,t}^{ESS,RAMPD} h^{RAMPD} \leq SOC_{i,\max}^{ESS} C_i^{ESS}, \forall i, t, \quad (27)$$

$$E_{i,t}^{ESS} = E_{i,t-1}^{ESS} + (P_{i,\alpha,t}^{ESS} \eta_{i,\alpha}^{ESS} - P_{i,\beta,t}^{ESS} / \eta_{i,\beta}^{ESS}) h, \forall i, t, \quad (28)$$

$$SOC_{i,\min}^{ESS} \leq E_{i,t}^{ESS} / C_i^{ESS} \leq SOC_{i,\max}^{ESS}, \forall i, t, \quad (29)$$

$$E_{i,0}^{ESS} = E_{i,T}^{ESS}, \forall i, \quad (30)$$

where  $\alpha_{i,t}^{ESS}$  and  $\beta_{i,t}^{ESS}$  are binary variables representing the working condition of the ESS  $i$  at time slot  $t$ .  $\alpha_{i,t}^{ESS} = 1, \beta_{i,t}^{ESS} = 0$  indicates the ESS is charging;  $\alpha_{i,t}^{ESS} = 0, \beta_{i,t}^{ESS} = 1$  indicates the ESS is discharging;  $\alpha_{i,t}^{ESS} = 0, \beta_{i,t}^{ESS} = 0$  indicates the ESS is standing by. The power limits of ESSs are shown in (21) and (22). In (23), the energy capacities provided by the ESSs are the difference between the discharging and charging power. In (24) and (25), the power limits of the ESSs are shown. Constraints (26) and (27) indicate that an ESS must be

able to maintain the fully deployed output level for  $h^E$  (typically 1 h) energy,  $h^{RES}$  (typically 1 h) spinning reserve, and  $h^{RAMP}$  (typically 15 min) ramping up/down [29]. Constraint (28) represents the relationship between stored energy and charging/discharging power. In (29), the ESS is bounded by the minimum and maximum of the state of charge (SOC). In (30), the initial and final stored energy should be equal.

### 3.2.4. The constraints of the microgrid

$$P_t^E = P_t^{WT,E} + P_t^{PV,E} + \sum_{i=1}^{N^{MT}} P_{i,t}^{MT,E} + \sum_{i=1}^{N^{ESS}} P_{i,t}^{ESS,E} - P_t^D, \forall t, \quad (31)$$

$$P_t^m = P_t^{WT,m} + P_t^{PV,m} + \sum_{i=1}^{N^{MT}} P_{i,t}^{MT,m} + \sum_{i=1}^{N^{ESS}} P_{i,t}^{ESS,m}, \forall t, m \in \{RES, RAMP, RAMPD\}, \quad (32)$$

The MG's capacity for energy and AS is supported by the DERs operated by the MG aggregator, as shown in (31) and (32). Therefore, the objective (7) and the constraints (8)–(32) form the proposed bidding model. The solution algorithm is elaborated in Section 5.

## 4. Reformulation via robust optimization approach

Because of its flexibility, controllability and moderate computational cost, the RO approach provides applicable solutions to the general stochastic optimization problems. In the optimal bidding model, the set of random variables includes the available wind and photovoltaic power. As elaborated in the Appendix A, this problem can be formulated as an RMILP by introducing dual and auxiliary variables as follows:

$$\max_{\Phi^E, \Phi^D} \sum_{s=1}^{N^S} \gamma_s (R_s^E + R_s^{RES} + R_s^{FRP} - C^{OP}) \quad (33)$$

subject to Constraints (14)–(32),

$$P_t^{WT,E} + P_t^{WT,RES} + P_t^{WT,RAMP} + \Gamma_t^{WT} z_t^{WT} + q_t^{WT} \leq \frac{1}{2} (\underline{P}_t^{AWT} + \bar{P}_t^{AWT}), \forall t, \quad (34)$$

$$P_t^{PV,E} + P_t^{PV,RES} + P_t^{PV,RAMP} + \Gamma_t^{PV} z_t^{PV} + q_t^{PV} \leq \frac{1}{2} (\underline{P}_t^{APV} + \bar{P}_t^{APV}), \forall t, \quad (35)$$

$$z_t^{WT} + q_t^{WT} \geq \frac{1}{2} (\bar{P}_t^{AWT} - \underline{P}_t^{AWT}) y_t^{WT}, \forall t, \quad (36)$$

$$z_t^{PV} + q_t^{PV} \geq \frac{1}{2} (\bar{P}_t^{APV} - \underline{P}_t^{APV}) y_t^{PV}, \forall t, \quad (37)$$

$$y_t^{WT}, y_t^{PV} \geq 1, \forall t, \quad (38)$$

$$z_t^{WT}, z_t^{PV}, q_t^{WT}, q_t^{PV} \geq 0, \forall t, \quad (39)$$

where  $z_t^{WT}, z_t^{PV}, q_t^{WT}, q_t^{PV}$  are the dual variables of the original problems and  $y_t^{WT}, y_t^{PV}$  are the auxiliary variables that help linearize the problem.  $\Gamma_t^{WT}$  and  $\Gamma_t^{PV}$  are the robustness parameters, which take on values in the interval  $[0, |J_t^{WT}|]$  and  $[0, |J_t^{PV}|]$ , where  $J_t^{WT}$  and  $J_t^{PV}$  are sets including all random variables in constraints (12) and (13) at time slot  $t$ . Note that there is just one random variable in each constraint, so  $|J_t^{WT}| = |J_t^{PV}| = 1$ . It is worth mentioning that by varying  $\Gamma_t^{WT} \in [0, |J_t^{WT}|]$  and  $\Gamma_t^{PV} \in [0, |J_t^{PV}|]$ , the flexibility of adjusting the robustness is acquired against the level of conservatism of the solution [33]. In practice, the robustness parameters

can be set according to the risk preference of the MG operator. The details of the RO approach can be found in [30].

The RMILP seeks to maximize the MG's revenues from each market under the worst case caused by the uncertainties in renewable generations. The hourly parameters  $\Gamma_t^{WT}$  and  $\Gamma_t^{PV}$  adjust the robustness degree of constraints (12) and (13) against the uncertainties in the wind and photovoltaic power. The larger the robustness parameters are, the more conservative the RMILP problem becomes. The influences of selecting different robustness parameters are investigated in Section 6.

## 5. Case studies

The test environment is a ThinkPad T440p operating at 2.40 GHz with 8 cores. The program is developed using MATLAB R2015a. The optimization solver is CPLEX 12.4 [34].

### 5.1. Basic data

Historical data of the Electric Reliability Council of Texas (ERCOT) day-ahead market prices [35] from July 1, 2016, to September 30, 2016, are used to generate 20 typical scenarios to address the uncertainties in day-ahead market prices. These price scenarios are generated by K-means clustering. The average hourly prices in the energy and AS markets are shown in Fig. 4. The wind and solar power are the real-world data from a wind farm and a photovoltaic station in a province in China. According to the uncertainty modeling in Section 3, under 95% confidence level, the forecasted wind and photovoltaic power and the confidence intervals are shown in Fig. 5. The parameters of the other DERs in the MG are shown in Table 1.

To evaluate the ramping capabilities and the benefits of the MG in joint energy and AS markets, three cases are considered:

**Case 1:** (i) S1, where the MG bids in the joint energy, reserve and FRP markets with  $\Gamma_t^{WT} = \Gamma_t^{PV} = 1$ ; (ii) S2, where the MG bids in the joint energy and reserve markets with  $\Gamma_t^{WT} = \Gamma_t^{PV} = 1$ ; (iii) S3, where the MG only bids in the energy market with  $\Gamma_t^{WT} = \Gamma_t^{PV} = 1$ .

**Case 2:** (i) S1; (ii) S4, where the MG bids in the joint energy, reserve and FRP markets with  $\Gamma_t^{WT} = \Gamma_t^{PV} = 0.6$ ; (iii) S5, where the MG bids in the joint energy, reserve and FRP markets with  $\Gamma_t^{WT} = \Gamma_t^{PV} = 0.2$ .

**Case 3:** i) S1; ii) S6, where the MG bids in the joint energy, reserve and FRP markets with  $\Gamma_t^{WT} = \Gamma_t^{PV} = 1$ , while the FRP prices

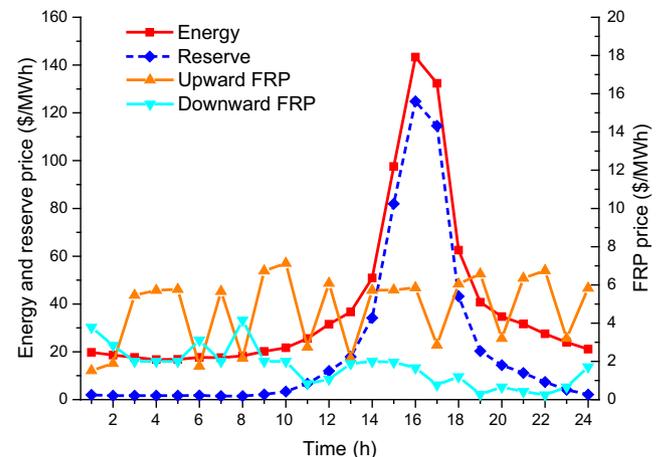


Fig. 4. Average day-ahead hourly prices in the energy and AS markets.

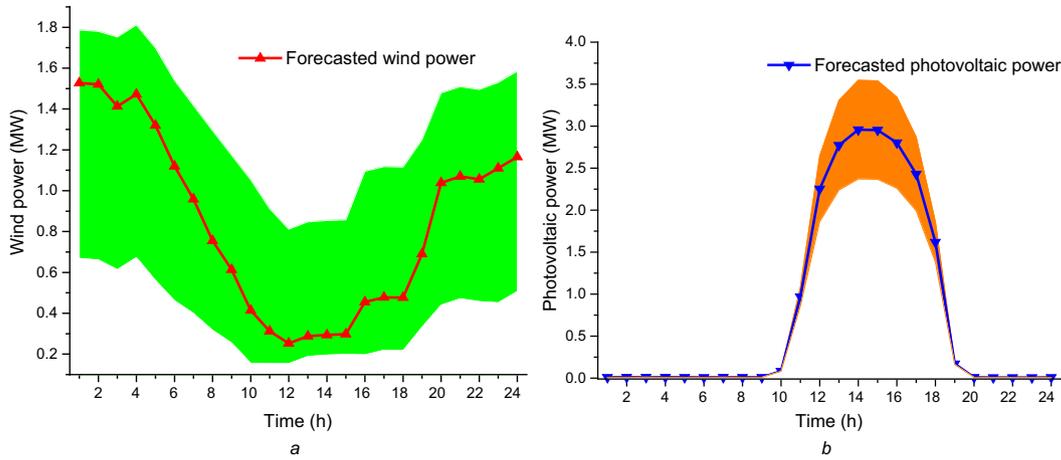


Fig. 5. The forecasted renewable power and the confidence intervals.

Table 1  
Parameters of the DERs in the MG.

DER	$c_i^{MT}$ (\$/MW h)	$P_{i,max}^{MT}$ (MW)	$P_{i,max}^{MT,RAMPUP}$ (MW/h)	$P_{i,max}^{MT,RAMPD}$ (MW/h)
MT-1	13	3	2	2
MT-2	10	4	2	2
MT-3	18	2	2	2
ESS-1	$\eta_{i,\alpha}^{ESS}$	$\eta_{i,\beta}^{ESS}$	$SOC_{i,min}^{ESS}$	$SOC_{i,max}^{ESS}$
	0.95	0.95	0.1	0.9
	$C_i^{ESS}$ (MW h)	$E_{i,0}^{ESS}$ (MW h)	$P_{i,\alpha,max}^{ESS}$ (MW)	$P_{i,\beta,max}^{ESS}$ (MW)
ESS-2	12	6	2	2
	$\eta_{i,\alpha}^{ESS}$	$\eta_{i,\beta}^{ESS}$	$SOC_{i,min}^{ESS}$	$SOC_{i,max}^{ESS}$
	0.95	0.95	0.1	0.9
	$C_i^{ESS}$ (MW h)	$E_{i,0}^{ESS}$ (MW h)	$P_{i,\alpha,max}^{ESS}$ (MW)	$P_{i,\beta,max}^{ESS}$ (MW)
	20	10	2.5	2.5

are 1.2 times of those in S1; iii) S7, where the MG bids in the joint energy, reserve and FRP markets with  $\Gamma_t^{WT} = \Gamma_t^{PV} = 1$ , while the FRP prices are 0.8 times of those in S1.

5.2. Base case results

In S1, the MG bids in joint energy, reserve and FRP markets. The optimal bidding strategies of the MG are shown in Fig. 6.

The MG strategically allocates the available capacity in each hour to maximize the revenues from the joint energy and AS markets. The bidding strategy of the MG depends on the physical constraints of the DERs and the opportunity costs in each market. In the energy market, the MG will generate electricity for the bulk power system, which leads to the operational costs of the MG. In the AS markets, the MG will leave a margin for the AS capacities, which may not cause the operational costs. As shown in Fig. 4, when the prices of spinning reserve are high and the opportunity costs are relatively low, the capacity is provided for reserve instead of bidding in the energy market. Similar conclusions can be drawn from the bidding for the upward FRPs. In addition, by curtailing renewable generation, decreasing the MTs' output and making the ESSs charge, the MG is able to provide downward FRPs.

The optimal bidding strategies of the DERs in the energy market are shown in Fig. 7. Because the operational costs of wind and photovoltaic power are zero, the capacity of WT and PV is fully used in the energy market to maximize the energy revenues. The operational cost of M-2 is relatively low; thus, all the available capacity is provided for energy. However, the costs of the other two MTs are higher, thereby driving MT-1 and MT-3 to bid the available capacity for ancillary services during some periods. In the process of

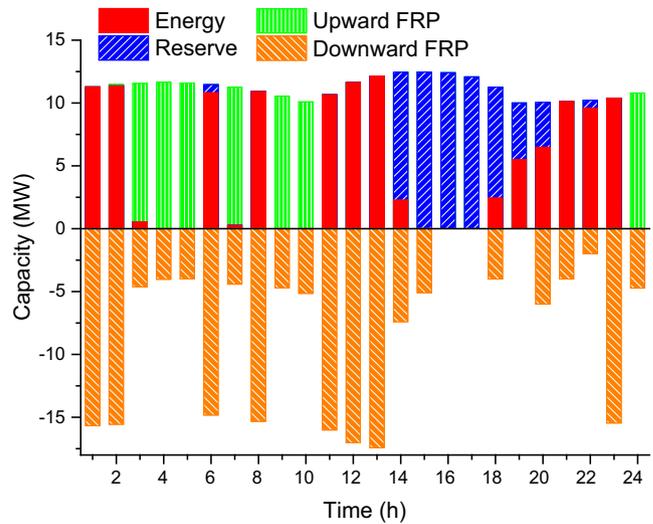


Fig. 6. Optimal bidding strategies of the MG in the base case.

arbitrage, the ESSs will charge during the valley hours and discharge during the peak hours. In addition, because the ESSs can flexibly adjust the consumption or production, the ESSs will strategically bid for energy and ancillary services. The energy capacity of the MG is equal to the difference between the capacity offered by the DERs and the load demands.

The expected revenues of the DERs in different markets are shown in Table 2. By strategically allocating the capacity of the

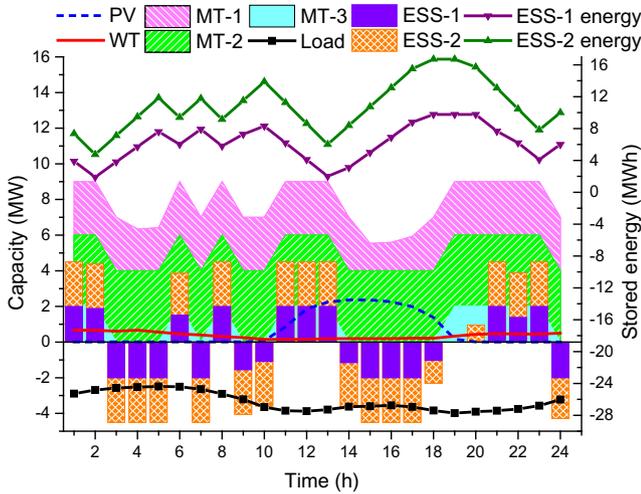


Fig. 7. Optimal bidding strategies of the DERs in the energy market.

**Table 2**  
The expected revenues of the DERs in different markets.

Revenue	Energy (\$)	Reserve (\$)	FRP (\$)	Total revenue (\$)
WT	275.99	0	15.01	291.00
PV	1189.46	0	16.71	1206.17
MT-1	1470.21	419.02	78.30	1967.53
MT-2	2805.89	0	75.68	2881.57
MT-3	257.99	797.51	123.88	1179.38
ESS-1	-534.68	1598.88	235.74	1299.94
ESS-2	-687.50	2014.55	301.70	1628.75
MG	1450.18	4829.96	847.02	7127.16

DERs in different markets, the MG can obtain the optimal expected revenues with 20.35% energy, 67.77% reserve and 11.88% FRP. As one can observe, it is beneficial for the MG to participate in the joint energy and AS markets, in which FRPs are also important fractions.

5.3. Comparison results in Case 1

Table 3 shows the expected revenues from each market in Case 1.

Comparing the results in S1 with those in S2, one can observe that the MG can increase its revenues by 8.02% if providing FRPs. Comparing the results in S1 and those in S3, one can observe that the MG can increase its revenues by 24.75% if participating in joint energy and AS markets.

Therefore, by participating in joint energy, reserve and FRP markets, the MG can further increase its revenues from the day-ahead markets. Meanwhile, the MG is able to provide ramping capacities for the bulk power system, which fully utilizes the grid-friendly potentials of the MG.

5.4. Comparison results in Case 2

The total FRPs provided by the MG in Case 2 are compared in Table 4.

**Table 3**  
The expected revenues from each market in Case 1.

Revenue	Energy (\$)	Reserve (\$)	FRP (\$)	Total revenue (\$)
S1	1450.18	4829.96	847.02	7127.16
S2	1498.84	5099.03	0	6597.87
S3	5713.36	0	0	5713.36

**Table 4**  
The total FRPs provided by The MG in Case 2.

	S1	S4	S5
Upward FRP (MW)	76.79	77.82	78.44
Downward FRP (MW)	187.42	190.46	193.93

**Table 5**  
The expected revenues from each market in Case 2.

Revenue	Energy (\$)	Reserve (\$)	FRP (\$)	Total revenue (\$)
S1	1450.18	4829.96	847.02	7127.16
S4	1605.92	4946.87	860.27	7413.06
S5	1801.27	5025.54	871.46	7698.26

From the comparison results, the total FRPs provided by the MG increase with the decrease of the conservatism degree. A smaller conservatism degree indicates a larger amount of available renewable generation is expected, thereby leading to an increase in the ramping capacities of the MG. The expected revenues from each market in Case 2 are shown in Table 5.

With the decrease of the conservatism degree, the revenues from each market will increase. As the simulation results show, with the synergy of the DERs in the MG, the renewable generation can cooperate with the MTs and ESSs and be fully accommodated without curtailment.

5.5. Comparison results in Case 3

The FRPs provided by the MG in Case 3 are compared in Fig. 8. From the comparison results, when the ramping capacities of the bulk power system are insufficient, leading to higher FRP prices, the MG is able to provide more ramping capacities to support the bulk power system while maximizing its individual revenues.

The expected revenues from each market in Case 3 are shown in Table 6. From the results in S1 and S6, by increasing the FRP prices by 20%, the total revenues of the MG from the day-ahead market will increase by 2.40%, and the FRP revenues increase by 21.77%. With more available capacities provided for FRPs, the revenues from energy and reserve are reduced in S6. From the results in S1 and S7, by reducing the FRP prices by 20%, the total revenues will decrease by 2.27%, the FRP revenues decrease by 26.30% while more capacities can be provided for energy and reserve in S7.

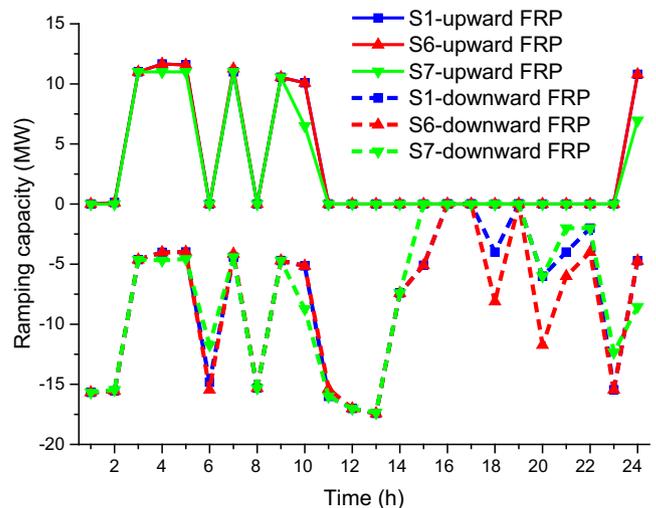


Fig. 8. The FRPs provided by the MG in Case 3.

**Table 6**  
The expected revenues from each market in Case 3.

Revenue	Energy (\$)	Reserve (\$)	FRP (\$)	Total revenue (\$)
S1	1450.18	4829.96	847.02	7127.16
S6	1444.17	4822.50	1031.38	7298.05
S7	1482.12	4858.81	624.27	6965.20

5.6. Sensitivity analysis

The RO approach adopted in this paper offers full control on the degree of conservatism for the constraints with uncertain coefficients. To demonstrate the influences of the degree of conservatism on the bidding strategies, the approach in this paper with different robustness parameters is compared with the method proposed in [36], in which the worst-case and most conservative scenario is considered.

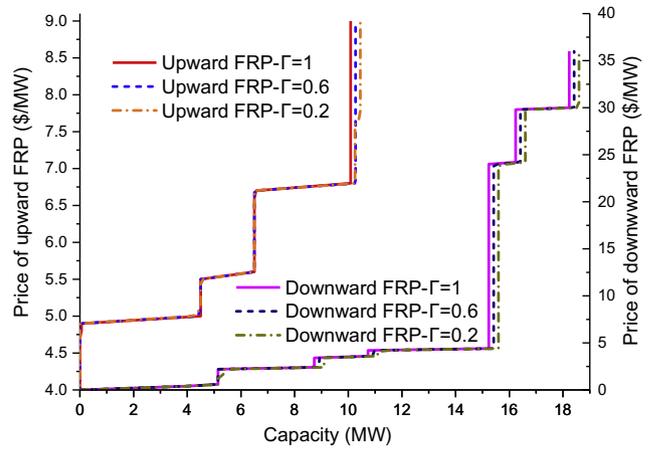
The revenues of the MG with different robustness parameters are shown in Fig. 9(a), and the growth rates of the revenues are shown in Fig. 9(b) compared with the method in [36].

As shown in Fig. 9 (a), with the increase of the conservatism degree, the revenues from each market are gradually reduced because of the decreasing expectations of the renewable generation. As shown in Fig. 9 (b), because the operational costs of the WT and PV are zero, most renewable energy will be allocated to the energy market, leading to the great change of the energy revenues with different robustness. Compared with the method in [36], the conservatism of the bidding problem can be flexibly adjusted by using the approach in this paper. Note that when the robustness parameters are equal to 1.0, two methods achieve the same revenues because the worst-case scenario is modeled and considered in the method in [36].

Then the base case S1 is simulated with different ramping prices to investigate the ramping capabilities of the MG under different levels of prices. With the prices of energy and reserve unchanged, the bidding curves of upward and downward FRPs provided by the MG at 10:00 are shown in Fig. 10.

From the bidding curves of FRPs, when the prices go up, the FRPs provided by the MG will increase. The maximal ramping capacities of the MG at 10:00 with different robustness parameters are shown in Table 7.

Under different degrees of conservatism, the bidding curves of the MG for FRPs have similar shapes. However, with the decrease of the degree of conservatism, the maximal ramping capacities of the MG will increase because more renewable generation is expected.



**Fig. 10.** The bidding curves of upward and downward FRPs provided by the MG at 10:00.

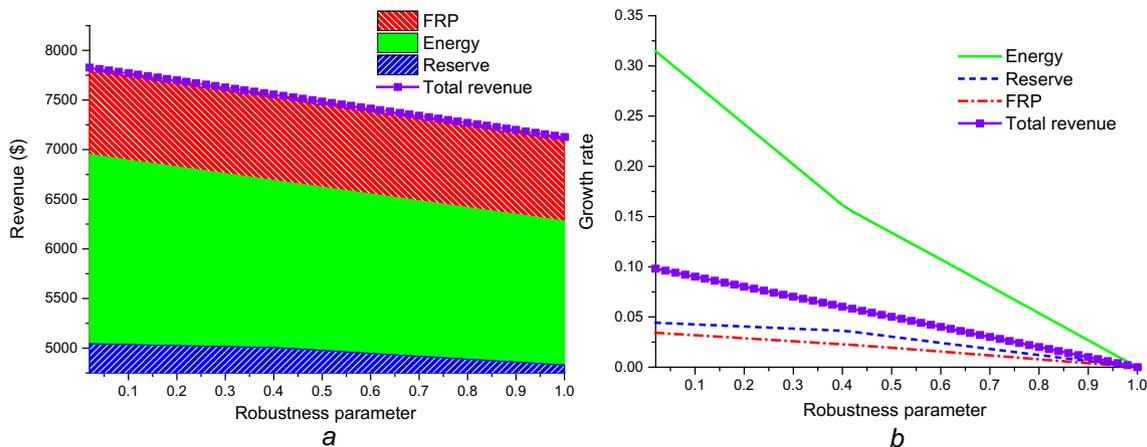
**Table 7**  
The maximal ramping capacities of the MG at 10:00.

	$\Gamma = 1$	$\Gamma = 0.6$	$\Gamma = 0.2$
Upward FRP (MW)	10.09	10.27	10.45
Downward FRP (MW)	18.24	18.42	18.60

6. Conclusions

Integrated with rapid ramping DERs, an MG is a promising energy resource to provide flexible ramping products for the power system. In this paper, flexible ramping products are incorporated in an optimal bidding framework for MGs for the first time. The bidding model aims at maximizing the expected revenues from the day-ahead energy, reserve and FRP markets. To address the uncertainties in renewable energy and day-ahead market prices, a hybrid stochastic/robust optimization approach is adopted. The bidding problem with uncertain coefficients can be transformed into a mixed-integer linear programming model.

Case studies based on an MG with various DERs demonstrate the market behavior of the MG using the proposed bidding model. According to the simulation results, 1) Compared with the cases where the MG only bids in the energy market and bids in energy and reserve markets, the MG is able to increase the revenues when



**Fig. 9.** The revenues and the growth rates of the MG with different robustness parameters.

participating in joint energy, reserve and FRP markets by 24.75% and 8.02%, respectively. 2) A smaller conservatism degree indicates that a larger amount of available renewable generation is expected, thereby leading to an increase in the ramping capacities of the MG. 3) By increasing the FRP prices by 20%, the total revenues of the MG will increase by 2.40%, and more available capacities will be provided for FRPs; By reducing the FRP prices by 20%, the total revenues of the MG will decrease by 2.27%, while more capacities will be provided for energy and reserve instead.

By incorporating FRPs in the bidding framework of the MG, on one hand, the MG is able to increase the revenues from market bidding; on the other hand, the power system will benefit from the improvement of the dispatch flexibility and the adequacy of the ramping resources. The proposed model will provide new insights in the development of MGs.

## Appendix A

A general robust optimization problem involving uncertain right-hand-side coefficients  $b_i$  is expressed as follows:

$$\max_{x_j, \forall j} \min_{b_i, \forall i} \sum_j c_j \cdot x_j \quad (40)$$

subject to

$$\sum_j a_{ij} x_j \leq b_i, \forall i \quad (41)$$

$$x_j \leq \bar{x}_j, \forall j \quad (42)$$

where  $x_j, \forall j$  represents the decision variable.  $\underline{x}_j$  and  $\bar{x}_j$  are the lower and upper bounds of  $x_j$ .  $c_j, \forall j$  and  $a_{ij}, \forall i, j$  are constant. Uncertainties only affect right-hand-side coefficients  $b_i$  of the inequality constraints.  $b_i$  is a random variable taking values in the interval  $[\underline{b}_i, \bar{b}_i]$ , where  $\underline{b}_i$  and  $\bar{b}_i$  are the lower and upper bounds of  $b_i$ . Then the problem can be transformed as follows:

$$\max_{x_j, \forall j} \sum_j c_j \cdot x_j \quad (43)$$

subject to

$$\sum_j a_{ij} x_j + z_i \Gamma_i + q_i - \frac{1}{2} (b_i + \bar{b}_i) \leq 0, \forall i \quad (44)$$

$$z_i + q_i + \frac{1}{2} (b_i - \bar{b}_i) y_i \geq 0, \forall i \quad (45)$$

$$z_i, q_i \geq 0, \forall i \quad (46)$$

$$y_i \geq 1, \forall i \quad (47)$$

$$x_j \leq \bar{x}_j, \forall j \quad (48)$$

where  $z_i$  and  $q_i$  are the dual variables of the original problem while  $y_i$  is an auxiliary variable.  $\Gamma_i \in [0, 1]$  is the robustness parameter which is used to adjust the degree of conservatism. The larger the robustness parameter  $\Gamma_i$  is, the higher degree of conservatism is. More details can be found in [30].

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