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A joint selfish routing and channel assignment game in wireless mesh networks

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Abstract

This paper designs a routing and channel assignment game – *Strong Transmission Game* in non-cooperative wireless mesh networks. Due to the nature of mesh routers (relay nodes), i.e., they are dedicated and have sufficient power supply, this game consists of only service requestors. Our main contributions in this paper are as follows: (1) We prove that there always exists a pure strategy Nash Equilibrium in the game and the optimal solution of our game is a Nash Equilibrium as well. (2) The price of anarchy is proved to be $\mathcal{O}(n^2)$. (3) Furthermore, our heuristic algorithms are introduced to approach the equilibrium state in the sense of the optimal routing and channel assignment response of every requestor, while the decisions from other agents are fixed. To evaluate our scheme, substantial simulation results are presented and the conclusion is twofold: (1) Our proposal is not far from the optimal. (2) Even performance gains can be expected, as compared with off-the-shelf techniques.

Keywords: Wireless mesh networks; Game theory; Nash Equilibrium; Selfish routing; Channel assignment

1. Introduction

Wireless mesh network (WMN) is a new configuration for wireless broadband networks. It is built on a mix of fixed mesh routers and mesh clients interconnected via wireless links. In some sense, WMN is an amendment to the mobile ad hoc network (MANET), it brings some advantages like reducing the installation costs, being capable of deployment on a large scale, increasing reliability and providing self-management [1]. The commercial applications are driven by these features. And some cities in the USA, such as Medford, Oregon, Chaska and Minnesota, etc., have deployed mesh networks [2].

Traditional MANET only consists of the users' devices which dynamically join the network, acting as both user terminals and routers for other devices. This router role requirement cannot be guaranteed, since obviously each user cares much more about its own device's battery power consumption than offering the relay service for other users. Naturally the users will turn off their facilities to save energy without regard to the other's requests. Some researchers believe this drawback mainly accounts for the failure of the civilian applications of MANET [18]. To overcome this flaw, the incentive-capable mechanism has been adopted in the previous research which results in a selfish routing problem among the relay nodes [17–19]. Nevertheless, the situation in WMN is much different, since the infrastructure of WMN type has shown its potential to become the mainstream in applications of WMN. The mesh routers in this mode are dedicated and always have sufficient energy supply. Furthermore, the topology seldom changes, and node failures are limited [3]. These

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superiorities make unnecessary of the previous selfish routing problem in infrastructure WMN, because no routers have to take care of the energy limitation there, and they have no previous charging role in the routing decision procedure. This ensures the success of multi-hop transmission in the infrastructure WMN.

However, service requestors cannot be satisfied so easily, because of their selfish nature, the reason is twofold: (1) they want to minimize the private cost for the service; (2) meanwhile they want to maximize the personal service performance. As long as the requestors share some common resources in WMN, competition among them cannot be avoided. Thus, some competition issues happen, and a game is born accordingly.

The competition issues have already caused selfish routing problems in the conventional wired network [12,13], but things are much different in the wireless network. The interference phenomenon is inherent in wireless networks; this phenomenon limits the capacity of the wireless network a lot. The multiple channels are powerful in alleviating the interference. To restrain the interference, the routing and channel assignments were always considered jointly in previous research [7,8,22]. This induces the requestors to combine their routing with channel assignment for the purpose of low-cost and efficient transmissions.

To sufficiently make use of the multiple channels, multiple radios are widely adopted in WMNs. By using multi-radio and multi-channel, nodes can transmit and receive simultaneously, or transmit to different nodes simultaneously. This advantage can be illustrated through the demonstration in Fig. 1. If there are four distinct channels (represented by the numbers 1, 2, 3 and 4) which are assigned as shown in the figure, and four radios on node C, then all the transmissions there can work concurrently without interference among them. But a hidden trouble which cannot be neglected is that the multiple transmissions may finally exceed the capacity of a multi-radio-equipped relay node. Then the danger of interference emerges. However, a paradox is always there, the low-cost choice for one requestor is also attractive to someone else, and then the congestion shows up and unexpectedly low performance occurs. So it is quite necessary for requestors to foresee such a hindrance and make a tradeoff between low-cost and high-performance based on the requirements.

In this paper, we study selfish routing problem in multi-radio multi-channel WMNs. Unlike the previous research on this topic, we consider that the requestors are selfish identified entities, who only care about their own costs paid for their successful transmissions and the corresponding performance. Game theory is appropriate for such a study. The game we discuss only is composed of service requestors who want to be served with both low costs and good performance. They can obtain their aims by cautiously deciding the route paths, and carefully adjusting the channel assignments along the paths. Bear in mind that the requestors are individual profit-maximizing entities. Naturally they will make decisions in a non-cooperative mode. At the same time, the feasibility of non-cooperation cannot be guaranteed all the time. Since some resources (like radios, communication media) must be shared among the transmissions belonging to different requestors, cooperation is necessary. Otherwise interference may come up unexpectedly. Then this factor should be treated carefully in the definition of the game.

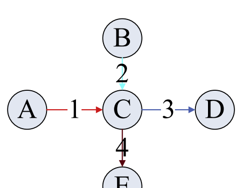


Fig. 1. A wireless mesh network with five nodes.

On the requestor side, all requestors obviously look forward to a valid ending a low-cost and low-interference one. The concept of Nash Equilibrium (NE) is a key role in this scenario where the requestors are selfish and non-cooperative. To formally define the NE, the corresponding factors should be selected cautiously. The concrete scheme should contain both cost factor (relative to the number of hops) and performance factor (like the interference level). Then equilibrium state can be achieved and accepted by all requestors.

Generally speaking, *If the Internet is the next great subject for Theoretical Computer Science to model and illuminate mathematically, then Game Theory, and Mathematical Economics more generally, are likely to prove useful tools* [9]. And game theory can implement the allocation scheme in a distributed manner with minimal communication overhead [4]. This article mainly focuses on the joint selfish routing and channel assignment problem in WMNs. The term relay node is used to represent the mesh router, and the source node is the mesh client who directly connects to some mesh router. The relay nodes gather the requestor mesh client nodes' requests and provide the access service for them. Our contributions are as follows.

- We formulate the selfish routing and channel assignment problem as a *Strong Transmission Game* by adopting an integrated model with an aim at solving this selfish problem in WMNs.
- We analyze this problem and prove the existence of a pure strategy Nash Equilibrium. Furthermore, the lower bound of the ratio between the equilibrium solution to the system optimum is settle to be 1, and the higher bound is proved to be $\mathcal{O}(n^2)$.
- We introduce our scheme and evaluate its outcome through simulations. The results show the difference between our scheme and the system optimum is not great; and compared to purely technical protocols, the selfish behaviors even bring some performance increases.

The rest of the paper is structured as follows. The related works are described in Section 2. This problem is defined and analyzed in Section 3. We introduce our algorithms in Section 4. In Section 5, we evaluate our scheme through simulations. Finally, a conclusion is in Section 6.

2. Related works

In recent years, an appreciable amount of research attention has been devoted to selfish routing. Selfish routing was initially introduced for studying and solving traffic-related issues, and especially for congestion problems in wired networks [12–16]. Gradually, ample research has already almost solved this problem in some sense. Also, some work has already been done in wireless networks. In the wired network, the major problem is congestion [5,6], however in wireless networks, the situation turns out to be more complicated for two reasons: (1) the relay nodes have no willingness to relay transmissions for others in MANET; (2) another added problem is interference along with the congestion.

To conquer the first problem, Luzi Anderegg and Stephan Eidenbenz introduced an economic solution to motivate the intermediate (relay) nodes, and use the VCG mechanism to ensure the truth and cost-efficiency of the routing [18]. Following this thinking, the truthful multicast routing was solved by Weizhao Wang, XiangYang Li and Yu Wang, without adopting the VCG mechanism [17]. The joint routing and forwarding problem was settled by Sheng Zhong, Li (Erran) Li, Yanbin Grace Liu and Yang Richard Yang via incentive-compatible and cryptographic techniques [19]. The participants in the above research are the relay nodes, which is obviously unreasonable, since the source nodes have an equal or even more important role in the system and no transmissions would exist if the source nodes were not satisfied with the charge provided by the relay nodes. Weizhao Wang, Stephan Eidenbenz, Yu Wang and Xiang-Yang Li dealt with this matter through game theory, especially, adopting Nash Equilibrium for the first time [20]. Recently, the collusion-resistant selfish routing is the newest progress in this research [21]. The common hypothesis of all these studies is the battery limitation of the relay node, which is true in MANETs but may not the same in the case of WMNs, due to the existence of dedicated and sufficient power supported mesh routers. Intuitively, without the second reason, the selfish routing problem in WMNs is much more like the scenario in wired networks. However, few papers consider the selfish routing problem combined with interference restriction.

Most efforts for the second problem are made in a centralized mode. Mansoor Alicherry, Randeep Bhatia and Li (Erran) Li studied the joint routing and channel assignment problem in multi-radio wireless mesh networks, with the aim to optimize the overall throughput of the system [7]. Richard Draves, Jitendra Padhye and Brian Zill considered the routing in multi-radio, multi-hop WMNs, and used the testbed to examine the effect of IEEE 802.11 a/b/g channel assignment [8]. Jian Tang, Guoliang Xue and Weiyi Zhang introduced a scheme of QoS routing in wireless mesh networks [22]. Nevertheless, these schemes do not take the nature of requestors' selfishness into account.

3. Our model and analysis

3.1. Preliminaries

We use an undirected graph $G(V,E)$ to model the wireless mesh networks, where V is the set of n wireless stationary nodes and E is the set of m edges. The undirected graph is sound for the assumption of 802.11 MAC protocol, and CSMA with RTS/CTS/ACK is used to protect transmissions. Bidirectional transmission is the necessary condition for this MAC protocol. Thus, the undirected graph is adopted to fulfill the bidirectional transmission requirement. There are a transmission radius $r > 0$ and an interference radius $R = q \times r (q \geqslant 1)$ associated with every node determined by the transmission power. We let $d(u,v)$ represent the Euclidean distance between u and v. Then there is an undirected edge $(u,v) \in E$ connecting node u and node v if $d(u,v) \leqslant r$. The edge (u,v) in G corresponds to a wireless link between node u and node v in the wireless network. Moreover, if there is interference between (u,v) and (u',v'), there is a necessary condition, $d(u,u') < R \bigvee d(u,v') < R \bigvee d(v,u') < R \bigvee d(v,v') < R$.

Transmission may collide in two ways in wireless networks: *primary* and *secondary interference* [23]. *Primary interference* occurs when a node has to transmit and receive simultaneously, or transmit/receive more than one packet on the same radio at the same time. *Secondary interference* occurs when a receiver node is just within the range of more than one sender node on the same channel. Half-duplex operation is enforced for each radio to prevent *primary interference*. That means one radio can only transmit or receive at one moment. *Secondary interference* is shown in Fig. 2, where the two transmissions work simultaneously on the same channel at the same time within the range of interference radius R.

3.2. Our radio duplex assumption

In fact, multi-radio multi-channel technology brings absolutely new features to the relay function. The second and third generation mesh networks use such advanced fea-

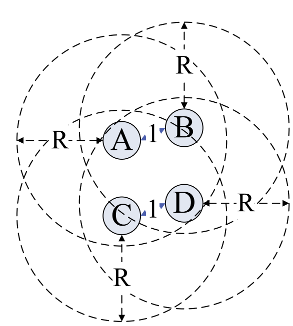


Fig. 2. An interference example.

tures in their routers (where client nodes have just one radio) [27]. The possibilities of both radio and channel assignment on the relay nodes in multi-radio scenario can be considered as follows. One possible case may be considered as demonstrated in Fig. 3a, where the radios can be assigned in an analogous method to traditional single radio relaying. Here, the relay node B uses only one radio (to relay by sequentially switching between the up and down links) to send the signal from node A to node C, while another radio is reserved for communication between another pair of source and sink nodes. We name this scheme "half duplex relay mode". Another possibility may be considered as depicted in Fig. 3b, where the relay node B uses a dedicated radio to receive the signal from node A, and meanwhile adopts another dedicated radio for concurrently sending the signal to node C. This is named "full duplex relay mode". The full duplex relay mode provides advantages such as higher bandwidth and lower time delay by keeping the channels fixed for A–B and B–C communications, unlike the half duplex mode, where the channel switching (if needed), as well as access control procedures, were essential. As the number of hops in the path increases, even more gain in performance can be achieved by carefully adjusting the channels on each edge along the path, which cannot be expected in the half duplex relay mode in the light of [28]. Also another advantage in the full duplex mode is that the available channels of each node can be re-used across the network. This leads to increase of spectrum and so also to improvement in the overall performance of the network.

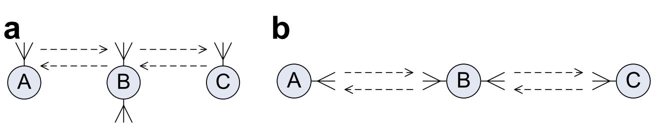


Fig. 3. Half and full duplex relay modes.

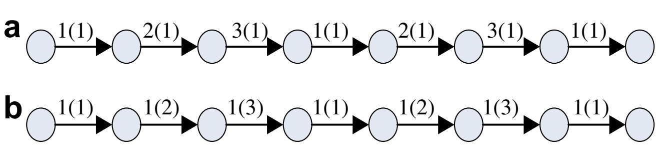


Fig. 4. Example of applying (a) half duplex and (b) full duplex modes in multi-hop paths.

For better realization of performance enhancement in full duplex mode in a longer multiple hop path, a simple example can be considered, as given in the Fig. 4. The graphs in Fig. 4a and b represent relay strategies for optimally attainable performance in half duplex and full duplex modes, respectively. There are six relay nodes between the source and the sink node. The first number on each edge denotes the time slot, and the second number in the bracket means the adopted channel. The corresponding simulation data from the NS2 are provided in Fig. 5 for performance evaluation.

It is evident from the curves in Fig. 5 that the full duplex relay mode wins overwhelmingly over the half duplex mode. Since the service requestors are all selfish in terms of minimizing their transmission monetary cost and maximizing performance, full duplex mode may suitably be accepted widely. Therefore, throughout this paper, it is assumed that all nodes adopt full duplex relay mode for their transmissions.

3.3. Strong transmission game

Here, we describe the following selfish routing problem in wireless mesh network. There are k source-sink node pairs $\{s_1, t_1\}, \ldots, \{s_k, t_k\}$ in $G(V, E)$, naturally, we denote the agent set $D = \{1, 2, \ldots, k\}$ according to $\{s_1, s_2, \ldots, s_k\}$ and denote the set of $s_i - t_i$ paths as P_i correspondingly. We let $A_i = P_i \bigcup\{\phi\} = \{\phi, p_i^1, p_i^2, \ldots, p_i^{n_i}\}$ be the set of all action available to agent i, which means the agent i is in charge of choosing a path p_i^j from s_i to t_i with feasible channel assignment, where j means an arbitrary jth action in agent i's action set, and define $P = \prod_i P_i$. For ease of exposition, we assume that each agent has the same CBR flow requirement. A vector $\mathrm{P} \in P$ is an action profile, and P_i is the ith element $\mathrm{P}_i \in P_i$. The number of hops with respect

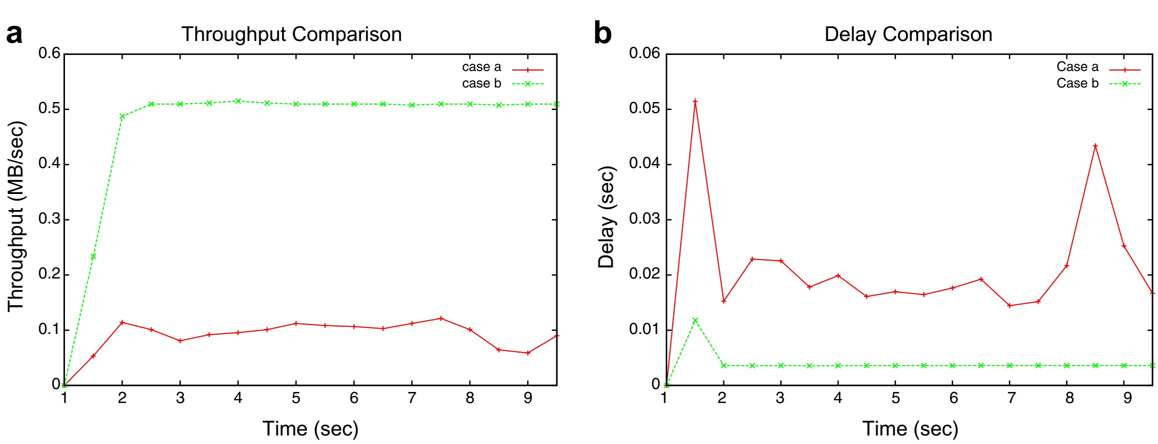


Fig. 5. Performance evaluation of half duplex and full duplex modes.

to an agent i is $h_i(\mathrm{P}_i)$. We represent the set of orthogonal channels by $OC = \{1, 2, \ldots, K\}, |OC| = K$. The maximum number of radios per node that can be used is denoted as Q, and the real number of radios per node used is a function $\gamma : V \to N$. A function $\zeta : V \to N$ represents the number of channels that are used or interfered with by a node.

Strong Transmission Game is a game where each agent can transmit simultaneously through a distinct path with feasible channel assignment without interference among them.

Here, distinct path means the routing path of any agent is a path with different radios from others' paths. This requirement can guarantee the non-primary-interference situation between any pair of agents.

We leave behind an open problem in this paper, that is how to determine an instance following the requirement of Strong Transmission Game or not. It is mainly caused by the complexity of the joint routing and channel assignment problem. In fact, even the pure channel assignment problem is an NP-complete issue [24]. To avoid being distracted to another topic, this problem will be settled in detail in other dedicated research. The obvious fact is shown as Lemma 1.

Lemma 1. *The Strong Transmission Game Determination problem is NP-complete.*

A surface flaw of our Strong Transmission Game is that it seems like a special case in WMNs. Actually, this can be extended to the common scenario by using some scheduling schemes. For example, if the requestors are too numerous to be served in the same period of time, they surely can be served in the Strong Transmission Game mode by turns through careful scheduling.

As we have mentioned earlier, from the natural point of view, two factors will be counted in the definition of private cost function: (1) the agents' costs for their transmissions and (2) their transmission performance. Although the relay nodes (mesh routers) in WMNs are dedicated, which means they cannot announce the price for the relay individually, they should gain some fixed amount of reward to cover their consumption for such relays. This leads to the monetary cost (factor 1) of each agent. Obviously, for each agent, this part can be measured by the number of routing path hops. The performance of an agent's transmission is affected significantly by the potential interference level when applying the CSMA/CA mechanism of the 802.11 protocol. We illustrate the potential interference level in Fig. 6.

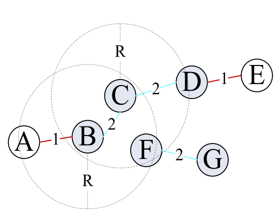


Fig. 6. A flow demonstration.

In Fig. 6, the link BC interferes with CD and FG. A data package will take three time slots to flow from node B to D (TDMA mode and using efficient scheduling to avoid interference). As a result, the throughput is reduced to one third that of the non-interference situation. Then we say the potential interference level of link BC is 3.

We use $\mathrm{IE}(e)$ to denote the potential interference level of link e on an arbitrary channel, and $\mathrm{IE}(e, i)$ to denote the potential interference level of link e on a specific channel i.

The traffic rate criterion can be omitted for the same CBR traffic assumption. Then integrating the two factors as a whole, we define the private cost function to agent i as $c_i(\mathrm{P}_i) = h_i(\mathrm{P}_i) \times \max \mathrm{IE}_{e \in \mathrm{P}_i}(e)$ and the social cost function $\kappa(\mathrm{P}) = \Sigma_i c_i(\mathrm{P}_i)$. Basically, the agents are selfish and rational, they will minimize their private costs while carefully adjusting the channel assignment to prevent mutual interference. That means an agent will minimize its $c_i(\mathrm{P}_i)$ subject to

$$\gamma(v) \leqslant Q \ \forall \ v \in \mathrm{P}_i \tag{1}$$

$$\varsigma(v) \leqslant K \ \forall \ v \in \mathrm{P}_i \tag{2}$$

and for any two agents i, j, $\forall e_a = (u_a, v_a) \in \mathrm{P}_i, \forall e_b = (u_b, v_b) \in \mathrm{P}_j$ on the same channel, it must be the case

$$d(u_a, u_b) > R \bigwedge d(u_a, v_b) > R \bigwedge d(v_a, u_b) > R \bigwedge d(v_a, v_b) > R \tag{3}$$

Eqs. (2) and (3) mainly assure no mutual primary and secondary interference between each pair of agents. "None mutual interference" is a necessary condition in the non-cooperative mode in Strong Transmission Game, because without cooperation, the selfishness will finally drive the agents to endless interference if the condition breaks the rules depicted by these two equations. Actually, interference between two links along one agent's routing path can be avoided efficiently, since all the links along this path are controlled by the same agent, so a feasible scheduling can be applied under the direction of this agent to eliminate interference.

We say that action profile $\mathrm{P} \in P$ is a pure strategy Nash Equilibrium if no agent has an incentive to change its action. That is, for any agent i

$$c_i(\mathrm{P}_i) \leqslant c_i(f(\mathrm{P}'_i)) \tag{4}$$

Here, P'_i represents the agent i changing its action (routing path or channel assignment or both), this new action is different from the previous one in P_i.

3.4. Main results

While dealing with a practical case of routing and channel assignment game, the pure strategy Nash Equilibrium is more appropriate rather than the mixed strategy one. This is because, each service requestor expects to be provided with a concrete path and channel assignment, without the time wasting problem of choosing an action amongst many on the basis of probability distribution.

Another important thing should be considered here, i.e., the existence of the pure strategy NE. In principle, the pure strategy equilibrium does not exist in all games, even though there is an NE (it is mixed strategy and exists in every game). So our next job is to prove whether there always exists a pure strategy NE in strong transmission game. This proof is given below.

Theorem 1. *In a Strong Transmission Game system* (κ, c_i), *there is a pure strategy Nash Equilibrium.*

Proof. We prove this theorem by the method of contradiction, through explicitly finding an action profile which is a pure strategy Nash Equilibrium for the Strong Transmission Game. Consider a directed graph $\mathcal{D}$, whose node corresponds to one action profile which is a feasible path, and, a corresponding channel assignment set. There is an arc from node $\mathrm{P}_0 = \{p_i, p_2, \ldots, p_i, \ldots, p_k\}$ (p_i represents the path and the channel assignment according to agent i) to node $\mathrm{P}_1 = \{p_i, p_2, \ldots, p'_i, \ldots, p_k\}$ if $c_i(\mathrm{P}_1) < c_i(\mathrm{P}_0)$ for some agent i. It follows that a node P_N in $\mathcal{D}$ corresponds to a pure strategy Nash Equilibrium if and only if the node has out-degree zero. So if $\mathcal{D}$ is acyclic and not empty, the system has a pure strategy Nash Equilibrium. Since the definition of the *Strong Transmission Game* can ensure $\mathcal{D}$ is not empty, we will prove that $\mathcal{D}$ is acyclic below.

Suppose $\mathcal{D}$ is not acyclic. Then take a directed cycle $\mathcal{C}$ in $\mathcal{D}$. Suppose the cycle contains nodes corresponding to the path sets: $\mathrm{P}_0 = \{p_1^0, p_2^0, \ldots, p_k^0\}, \mathrm{P}_1 = \{p_1^1, p_2^1, \ldots, p_k^1\}, \ldots, \mathrm{P}_t = \{p_1^t, p_2^t, \ldots, p_k^t\}$. Here, $\mathrm{P}_0 = \mathrm{P}_t$. It follows that the action profile P_r and P_{r+1} differ for one agent, say agent i_r. So $p_i^r = p_i^{r+1}$ if $i \neq i_r$ and $c_i(\mathrm{P}_{r+1}) < c_i(\mathrm{P}_r)$, therefore it must be the case that $\Sigma_{r=0}^{t-1} c_i(\mathrm{P}_{r+1}) - c_i(\mathrm{P}_r) < 0$. If the agent i_r changes its path or adjusts channel assignment, then Eqs. (2) and (3) can assure that the change will not affect any other agent's private cost. That means

$$c_i(\mathrm{P}_{r+1}) - c_i(\mathrm{P}_r) = \kappa_i(\mathrm{P}_{r+1}) - \kappa_i(\mathrm{P}_r) \tag{5}$$

Then, since $\mathrm{P}_0 = \mathrm{P}_t$, we obtain

$$\begin{aligned}\Sigma_{r=0}^{t-1} c_i(\mathrm{P}_{r+1}) - c_i(\mathrm{P}_r) &= \Sigma_{r=0}^{t-1} \kappa_i(\mathrm{P}_{r+1}) - \kappa_i(\mathrm{P}_r) \\ &= \kappa_i(\mathrm{P}_t) - \kappa_i(\mathrm{P}_0) = 0 \end{aligned} \tag{6}$$

That is a contradiction to $\Sigma_{r=0}^{t-1} c_i(\mathrm{P}_{r+1}) - c_i(\mathrm{P}_r) < 0$. So $\mathcal{D}$ is acyclic. Then Theorem 1 follows. □

The existence of pure strategy Nash Equilibrium leads us to the next step in theoretic thinking: What is the ratio between the pure strategy Nash Equilibrium and the system optimal solution? The following theorems will answer this question.

Theorem 2. *The system optimal solution is a Nash Equilibrium.*

Proof. Considering P_* as the optimal solution, it is always true that

$$\forall \mathrm{P}'_* \in \mathrm{P}, \quad \kappa(\mathrm{P}_*) \leqslant \kappa(\mathrm{P}'_*) \tag{7}$$

Then, if we assume that the optimal solution is not a Nash Equilibrium, this means that there exists some agent i who can reduce its private cost by changing its action unilaterally to a new one, and without producing interference to any other agent's routing path and channel assignment in existence (Eqs. (2) and (3)). Thus, he will not influence any other agent's private cost. Without loss of notational generality, this profile can be represented by P'_*.

Then $c_i(\mathrm{P}'_*) < c_i(\mathrm{P}_*)$ and we have $c_i(\mathrm{P}'_*) - c_i(\mathrm{P}_*) = \kappa(\mathrm{P}_*) - \kappa(\mathrm{P}'_*)$. Thus

$$\kappa(\mathrm{P}_*) > \kappa(\mathrm{P}'_*) \tag{8}$$

However, Eq. (8) is a contradiction of Eq. (7), then the assumption is wrong and the optimal solution is a Nash Equilibrium. □

Theorem 3. *The price of anarchy* [11] *is* $\mathcal{O}(n^2)$.

To prove this theorem, we introduce the following two lemmas first.

Lemma 2. *The upper bound of the ratio between any separate private routing cost* $(c_i = h_i(P_i))$ *to the optimal case is* $\mathcal{O}(n)$.

Proof. In Fig. 7, just thinking the routing path issue, let's assume each node has two radios and full duplex relay mode is adopted, there are two Nash equilibria: NE(u, x) and NE(v, y), so it is clear that to agent A, the upper bound of the cost ratio between the worse routing path and the optimal one is $\frac{c_A(u)}{c_A(v)} = \frac{h_A(u)}{h_A(v)} = \frac{n-10}{3} = \mathcal{O}(n)$. The lemma follows. □

Lemma 3. *The upper bound of the ratio between any separate private channel assignment cost* $(c_i = \max IE_{e \in P_i}(e))$ *to the optimal case is* $\mathcal{O}(n)$.

Proof. As mutual interference is forbidden by Eqs. (2) and (3), the potential interference level will just emerge along an agent's own routing path. This path is at most $(n-1)$ hops where all the n nodes construct this path. The worst case of

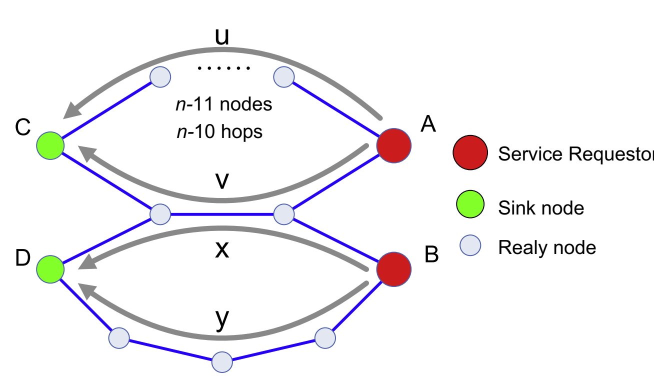


Fig. 7. A high routing ratio demonstration.

channel assignment is that all the $(n-1)$ hops are assigned to the same channel, where all of them can interfere with one another, i.e., $c_i(\mathrm{P}_i^{\mathrm{worst}}) = \max \mathrm{IE}_{e\in \mathrm{P}_i^{\mathrm{worst}}}(e) = n-1$. The optimal could be the case that all the $(n-1)$ hops are adjusted to the suitable channel, then $c_i(\mathrm{P}_i^{\mathrm{opt}}) = \max \mathrm{IE}_{e\in \mathrm{P}_i^{\mathrm{opt}}}(e) = 1$, thus the ratio is $\frac{c_i(\mathrm{P}_i^{\mathrm{worst}})}{c_i(\mathrm{P}_i^{\mathrm{opt}})} = \frac{\max \mathrm{IE}_{e\in \mathrm{P}_i^{\mathrm{worst}}}(e)}{\max \mathrm{IE}_{e\in \mathrm{P}_i^{\mathrm{opt}}}(e)} = \frac{(n-1)}{1} = \mathcal{O}(n)$. The lemma follows. □

Now, we prove Theorem 3 based on these two lemmas.

Proof. We use P^{w} and P^{o} to represent the worst case Nash Equilibrium, and the optimal solution, respectively. Thus, the price of anarchy can be denoted as: $\frac{\kappa(\mathrm{P}^{\mathrm{w}})}{\kappa(\mathrm{P}^{\mathrm{o}})}$. And by applying Lemma 2 and 3, we get:

$$\begin{aligned}
\frac{\kappa(\mathrm{P}^{\mathrm{w}})}{\kappa(\mathrm{P}^{\mathrm{o}})} &= \frac{\Sigma_i c_i(\mathrm{P}_i^{\mathrm{w}})}{\Sigma_i c_i(\mathrm{P}_i^{\mathrm{o}})} \leqslant \max \frac{c_i(\mathrm{P}_i^{\mathrm{w}})}{c_i(\mathrm{P}_i^{\mathrm{o}})} \\
&= \max \frac{h_i(\mathrm{P}_i^{\mathrm{w}}) \times \max \mathrm{IE}_{e\in \mathrm{P}_i^{\mathrm{w}}}(e)}{h_i(\mathrm{P}_i^{\mathrm{o}}) \times \max \mathrm{IE}_{e\in \mathrm{P}_i^{\mathrm{o}}}(e)} \\
&\leqslant \max \frac{h_i(\mathrm{P}_i^{\mathrm{w}})}{h_i(\mathrm{P}_i^{\mathrm{o}})} \times \max \frac{\max \mathrm{IE}_{e\in \mathrm{P}_i^{\mathrm{w}}}(e)}{\max \mathrm{IE}_{e\in \mathrm{P}_i^{\mathrm{o}}}(e)} \\
&= \mathcal{O}(n) \times \mathcal{O}(n) = \mathcal{O}(n^2)
\end{aligned} \tag{9}$$

Then, Theorem 3 follows. □

4. Algorithms

Frankly speaking, the Nash Equilibrium is the most suitable result to measure a non-cooperative game. However, one barrier that cannot be eliminated is the complexity of the Nash Equilibrium finding problem [25]. Even the pure strategy Nash Equilibrium is difficult to converge in a game with general pure strategy Nash Equilibrium [26]. Obviously the heuristic search is a preferred measure of our scheme. The key criteria to design the algorithms is that the outcome of algorithms should be a representation of an acceptable end for selfish agents. Naturally, the agents would feel the end is acceptable, when they cannot improve their routing path or channel assignment by changing their action unilaterally. Following this consideration, we introduce algorithms in this section.

At first, the Algorithm 1 is designed to provide an equilibrium state for all agents who use the number of hops to determine their routing paths. Surely, the conflict of using the same radio at the same time is not acceptable for them. And since in Section 3 we justified that selfish agents would apply full duplex relay mode, the nodes in Algorithm 1 would have an even number of radios. This assumption is necessary for the agents to escape from plunging into a ridiculous subgame for the one redundant radio on some relay node where this node has an odd number of radios.

Algorithm 1 Minimum hop-count path algorithm

Require: Initiate graph $G(V,E)$ and the agent array $D = \{1,2,\dots k\}$, $\mathrm{P} = \emptyset$.
for each agent $i \in D$ **do**
  Apply Dijkstra algorithm to compute an $s-t$ path p_i in G with minimum hop-count for agent i.
  if path p_i is feasible **then**
    $\mathrm{P} = \mathrm{P} \cup p_i$
    for $\forall v \in p_i$ **do**
      $\gamma(v) = \gamma(v) - 2$
      if $\gamma(v) == 0$ **then**
        $V = V/v$
      end if
    end for
  end if
end for
if each agent finds a feasible path **then**
  Output P
end if

The Algorithm 1 is a straightforward approach for the routing path problem. By using Dijkstra algorithm, each agent can be provided with a minimum hop routing path which will not intersect any other's path to the same radio. Thus, each agent has got a feasible path, and then he will assign the channel along his path to improve his performance next, as Algorithm 2 shows.

Unlike the routing algorithm, which in fact can guarantee a Pareto Equilibrium for the minimum hop routing path oriented selfish agents (because of the feature of Dijkstra algorithm), the channel assignment algorithm cannot completely assure an equilibrium state, due to the complexity of the channel assignment problem. The Algorithm 2 is designed for the link-oriented channel assignment optimization for the agents. That means each agent will improve his channel assignment by assigning some links along his routing path to different channels, if the new channels have lower potential interference levels than the previous ones. As this procedure in Algorithm 2 ends when all agents cannot improve their link-oriented channel assignments, this channel assignment can be regarded as a valid end of selfish behaviors.

The Formula 3 cannot be guaranteed by Algorithm 2 because mutual secondary interference may occur when the number of distinct channels is quite limited. To absolutely forbid this condition, the simplest way is just to let $K \geqslant k$. Thus, from the very beginning, there is no mutual secondary interference in Algorithm 2 (because each agent's routing path will be assigned to a distinct channel in the initiation phase), of course the situation is the same at the end.

5. Evaluation

In this section, we investigate the efficiency and results of our scheme through a simulated network scenario. To present a thorough evaluation of our approach, two objectives are studied: (1) The scheme analysis which focuses on cost issues, including price of anarchy, social cost and algorithm efficiency, etc. (2) The network performance results caused by our proposed selfish routing and channel assignment solution to the off-the-shelf techniques, measured by system throughput, loss rate, and transmission delay, etc. Now, we present these two topics.

5.1. Scheme analysis

In this subsection, we generated a wireless mesh network with 100 nodes as be shown in Fig. 8a. These nodes are randomly placed in an $800 \times 800\ \mathrm{m}^2$ rectangular region. Each node has two radios and $R = 2 \times r$. We vary the transmission range and the number of $s - t$ pairs which are chosen randomly from the 100 nodes in each round to evaluate their affection. Fig. 8b and c show the links where the maximum transmission ranges are no more than 140 m and 190 m respectively.

From Fig. 9a–f, we show some examples for our algorithms where the different line colors and styles represent different channels. Fig. 9a, c and e represent the end of Algorithm 1 where the numbers of $s - t$ node pairs are 2, 5 and 10, respectively. And b, d and f represent the end of Algorithm 2 analogically. The transmission ranges are all 140 m in these examples.

Algorithm 2. Channel assignment algorithm

Require: Initialize the channel set $OC = \{1, 2, \ldots, K\}$, the path array $\mathrm{P} = \{p_1, p_2, \ldots, p_k\}$ and according to potential interference level, $\forall p_i \in \mathrm{P}$, assign all link(s) along p_i to the channel $(i-1)\ mod\ K + 1$; use a number t to represent the current path, a number i to represent the current channel; set $channel\_flag = \mathrm{TRUE}$ to control the loop.

```
while channel_flag do
  channel_flag = FALSE
  for each channel i ∈ OC do
    for each path p_t ∈ P do
      Find the link e ∈ p_t that has the maximum
      IE(e) and satisfies IE(e,i) < IE(e), with also no
      mutual secondary interference. Assign this edge to
      channel i and update the potential interference level
      IE(e), ∀e ∈ p, ∀p ∈ P, set channel_flag = TRUE.
    end for
  end for
end while
```

The price of anarchy is a major criterion to evaluate the efficiency of the equilibrium solution of a game. Since the system optimal solution problem is NP-complete, we use a lower bound of the optimal solution instead of the optimal one to compute the price of anarchy when the number of $s - t$ pairs is 5 and 10. This lower bound is calculated under a scenario where each node has unlimited number of distinct channels (this lower bound scenario is not used when the number of $s - t$ pairs is 2. In that situation, enumeration is adopted).

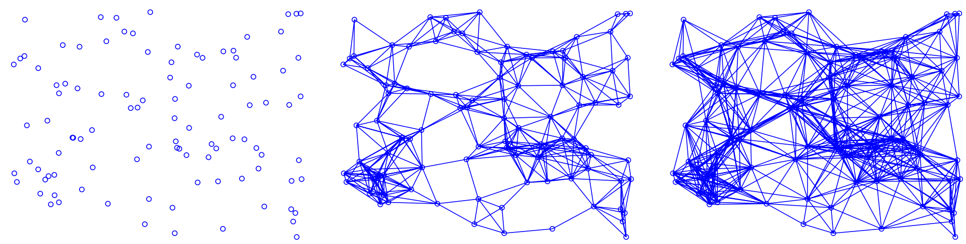
The Fig. 10a gives the simulation results of the price of anarchy. When there are two $s - t$ pairs, the enumeration is used to find the system optimal, compared to the enumeration result, ours can provide the same quality. It is because that there is almost no competition for these two agents to choose the relay nodes. Thus, they can use the minimum hop routing paths without any handicap. And by carefully assigning the channels, our scheme can guarantee the quality, too. As the $s - t$ pairs turn to 5 and 10 pairs, the price of anarchy is no longer 1 any more. And with more distinct channels, the price of anarchy is lower. This fact is reasonable since (1) the same lower bound scenario does good to the more distinct channels situation; (2) by using the added distinct channels, the agents can avoid some previous collisions. The slope of line from two $s - t$ pairs to five is steeper than the one from 5 to 10 is partly because the lower bound scenario is adopted since $s - t$ pairs are five, which is not the case in two $s - t$ pairs situation.

An important conclusion from this figure is: our scheme is not far from the system optimal. The fact is that our result is at most 134% of the system optimal. In reality, the extreme case is unusual, so commonly, our proposal can work well.

The relative between the transmission range and society cost with different number of orthogonal channels are illustrated in Fig. 10b–d. In these figures, the society cost decreases a lot as the transmission range is increased. It can be explained that the increase of transmission range brings out some new mediate nodes for the agents which can efficiently alleviate the collision and decrease the society cost. On the other hand, the number of distinct channels is the bottleneck for routing, especially in Fig. 10b. When the number of orthogonal channels increases, the society cost decreases dramatically in Fig. 10b. The reason is that the distinct channels are powerful even in one path scenario shown in Fig. 4. This will also be proved in the next subsection.

Another question about our scheme is about the convergence rate of the algorithm. As be shown in Fig. 10e, we fixed the number of $s - t$ pairs to 10, the number of steps of Algorithm 2 decreases as the transmission range increases since less hops are enough for a transmission. As more channels bring with more chances to alleviate collisions, the Algorithm 2 runs more rounds with more channels than the less channels situation.

At last of this subsection, we considered the question if the agent should pay for the transmission power consumption. Because the power consumption increases super-linearly as the transmission range increases, we use the square of transmission radius (divided by a constant) to replace the hop and the results are shown in Fig. 10f. This figure



(a) Nodes Deployment (b) r =140 (c) r =190

Fig. 8. A network with 100 nodes.

indicates that the total cost can hardly say to decrease as the transmission range increases, which means each agent should take care of the transmission range if the power consumption is taken into account.

5.2. Performance evaluation

In this subsection we simulate our proposed scheme in NS2 and from the output result sets, we draw evaluative inferences on the performance and efficiency of the model. From this simulation, we find the deterministic factors such as throughput, latency and loss rate for judging the performance of the overall performance of the network.

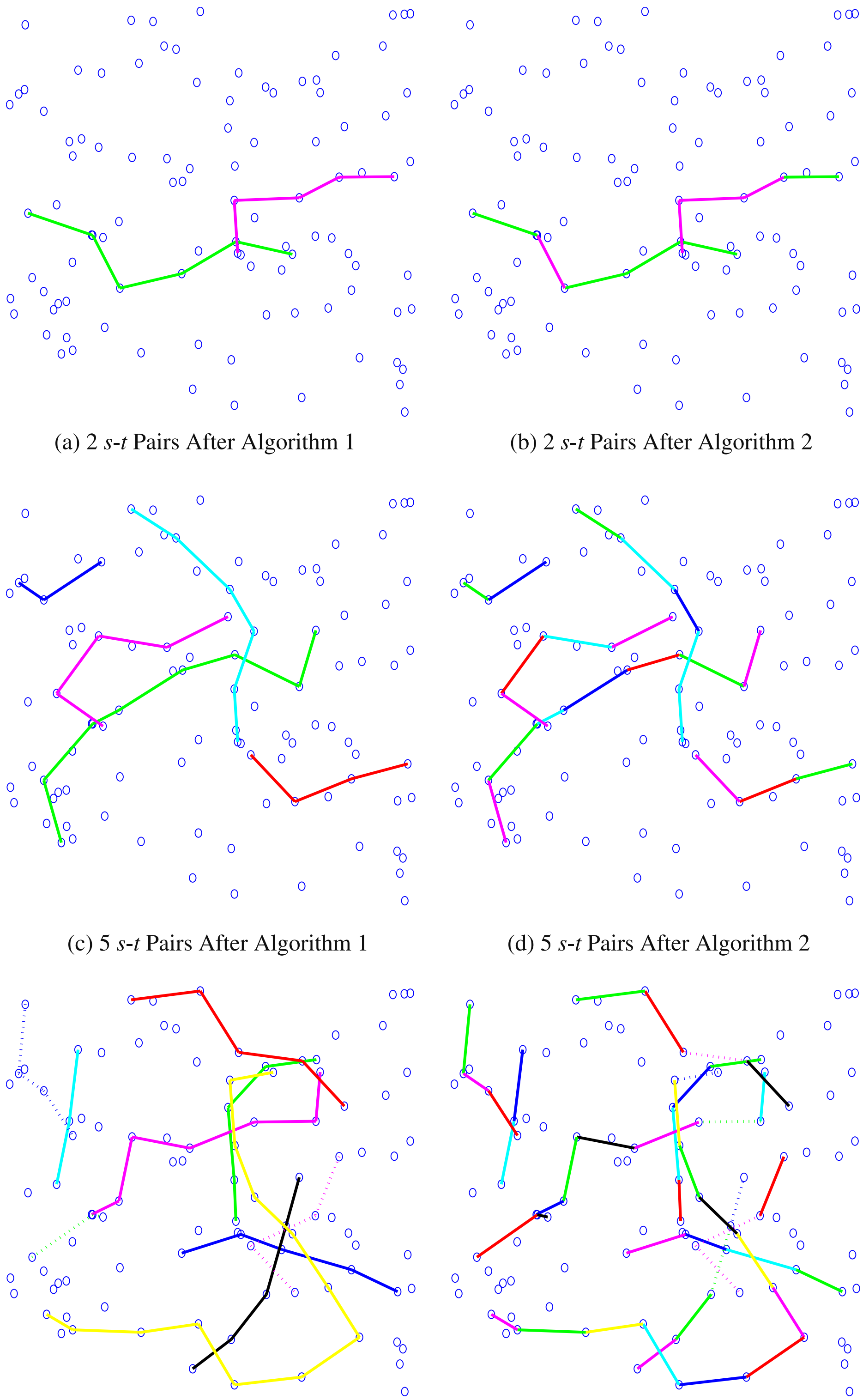
In order to simulate our network, 100 uniform terminals were randomly generated in an $800 \times 800\,\mathrm{m}^2$ region. The maximum transmission range of each terminal was adjusted to 150 m. The total number of $s - t$ pairs was fixed to be 10 and each relay node has two radios similar to the third generation off-the-shelf mesh router [27].

A main question in this simulation is whether selfish routing and channel assignment result in bad performance in Strong Transmission Game compatible WMNs environments. We try to answer this question through a thorough performance comparison to the SM scheme [29]. This scheme is chosen because it can follow the 802.11 MAC protocol without modification. The routing protocol adopts DSR as it is similar to our service requestor routing determination style, a source routing style. The solution of this scheme is denoted as SM and ours is ST in Figs. 11–13. The total distinct channel number is varied to 10, 15 and unlimited (represented as 10, 15 and U in Figs. 11–13) respectively. A traditional one channel based DSR protocol is taken into this simulation as the preliminary performance comparison calibration, denoted as DSR in Figs. 11–13. For each agent, a CBR flow is generated in this network. Each CBR flow has an offered load ranging from 100 to 1000 kb/s. Every radio can share all the distinct channels throughout this simulation.

The system aggregated throughput results are shown in Fig. 11. Our proposal configured with unlimited number of channels, namely ST-U, achieves the best performance. It is at least 2 per thousand and at most 169% better than SM-U under different load conditions. Compared to the ST-15 and ST-10, we can see ST-U has a 66% and 149% increase for ST-U, respectively. On the SM side, SM-U gets an increase ranged from 41% to 62% compared to SM-15 and an increase ranged from 63% to 90% compared to SM-10. The system throughput of ST-15 is worse than SM-15 when each flow's load is 100 kb/s, but as the load of each flow is raised, ST-15 can get an increase of 156% at most. The 10 channel situation is similar, ST-10 takes a lead about 82% to SM-10 at most. The original one channel based DSR protocol even cannot serve these requests and the aggregated throughput almost can be neglected as compared to the others. The primary fact justified by these curves is selfish behaviors bring throughput promotion to the off-the-shelf techniques in WMNs. It is reasonable since the selfishness of each agent will minimize his cost based on global information meanwhile the existing protocols always focus on the local information. The secondary conclusion can be drew is that ST can take more advantage of added spare channels than SM. This can be explained for the necessary of a common control channel in SM, thus the added spare channels cannot be efficiently utilized for the bottleneck of the common control channel.

Recall that we have mentioned in the previous subsection about the power of distinct channels. In Fig. 11, no matter ST or SM solution, more distinct channels bring remarkably higher system throughput. This performance evaluation is consistent with our scheme analysis.

The transmission delay data curves are illustrated in Fig. 12. Similar to the aggregated system throughput, the delay of scheme ST is always better than SM under 10, 15 and unlimited distinct channel conditions. The interesting thing is that the delay of ST-U is higher than ST-15 and ST-10 at all time. The ST-15 is at least 2% and at most 32% faster than ST-U. What's more, ST-10 is even faster than ST-15 ranged from 1% to 21%. It is the accumulated time due to the relay cost of the nodes where the signal receiving channel is different to the sending one. This phenomenon does not exist in the SM scheme where the delay is mainly caused by the common control channel, as the new spare channels are added in, the utilization ratio of the common channel is possibly decreased or even increased randomly, then SM-U is faster than SM-15 but slower than SM-10. The same thing in delay evaluation as the system



(a) 2 *s-t* Pairs After Algorithm 1

(b) 2 *s-t* Pairs After Algorithm 2

(c) 5 *s-t* Pairs After Algorithm 1

(d) 5 *s-t* Pairs After Algorithm 2

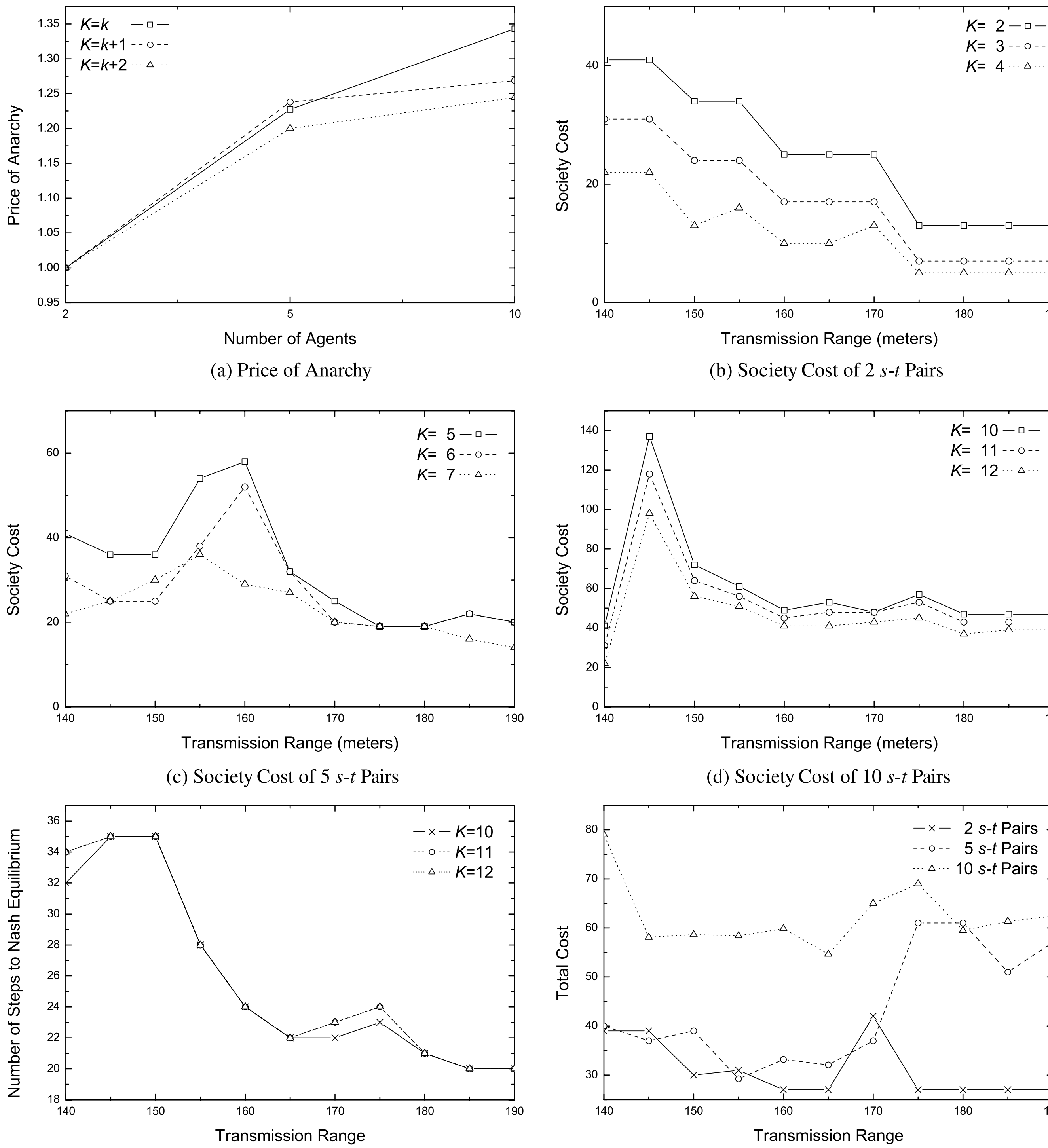
(e) 10 *s-t* Pairs After Algorithm 1

(f) 10 *s-t* Pairs After Algorithm 2

Fig. 9. Some routing and channel assignment results.

throughput is that the original one channel based DSR protocol has the highest delay which is not compared with the ST and SM schemes.

Fig. 13 exhibits the packet loss rate data curves. The ST scheme is also superior to the SM scheme under all conditions. One point should be paid attention is that higher loss rate occurs with more channels available in all most all conditions. This justifies the overhead for the utilization of the more distinct channels. Once again the original one channel based DSR protocol is the worst case without compare.



(a) Price of Anarchy
(b) Society Cost of 2 *s-t* Pairs
(c) Society Cost of 5 *s-t* Pairs
(d) Society Cost of 10 *s-t* Pairs
(e) Convergence Rate
(f) Total Cost With Power Consumption

Fig. 10. Simulation results.

6. Conclusions

We introduced an attempt to route flow in multi-channel, multi-radio wireless mesh network by using game theoretic tools in this paper; we proved the existence of pure strategy Nash Equilibrium in the proposed game. To guarantee the selfish agents' satisfactory, algorithms have been designed for this purpose. and justified to be feasible through extensive simulations. The feasibility of our scheme is supported by two evaluation facts: (1) Our result is close to the optimal measured by the price of anarchy. (2) Even performance gains can be expected as compared to the off-the-shelf techniques. In reality, the local optimization mechanism limits the off-the-shelf techniques a lot meanwhile the global point of view help the agents a lot even they behave in a selfish manner.

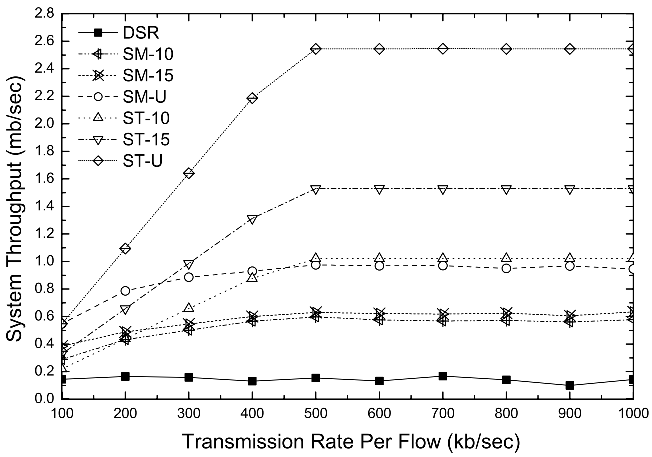


Fig. 11. Comparison of overall throughput of the network.

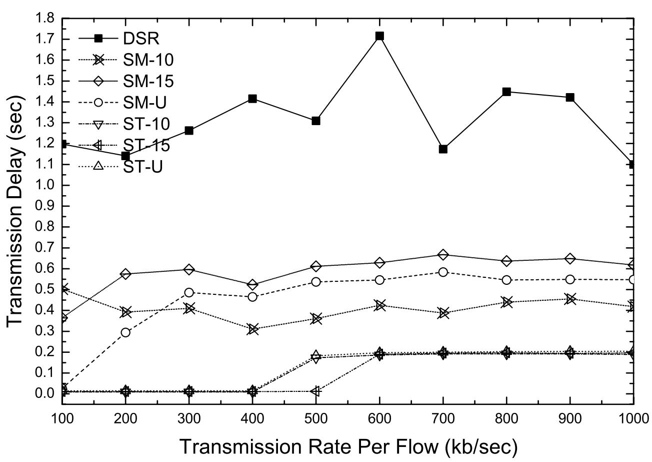


Fig. 12. Comparison of overall latency of the network.

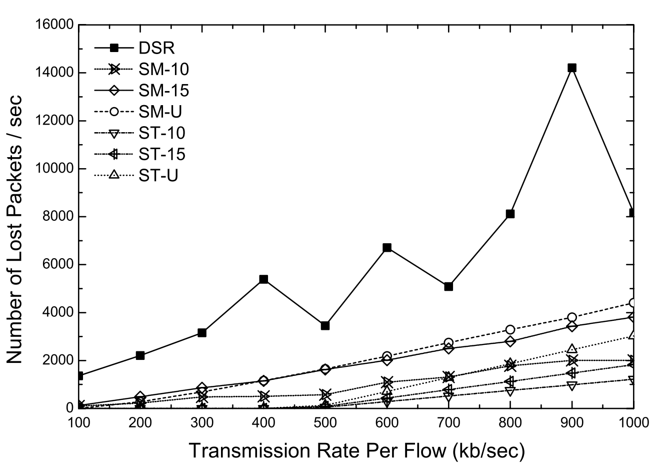


Fig. 13. Comparison of lost packet rate across the network.

As we have mentioned previously, this Strong Transmission Game can be extended to the general case by using a scheduling scheme. But like the routing and channel assignment problem itself, the scheduling problem will also result in a game among these agents. To improve our present research, a thorough study of the scheduling contained game is quite necessary in our future work.

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