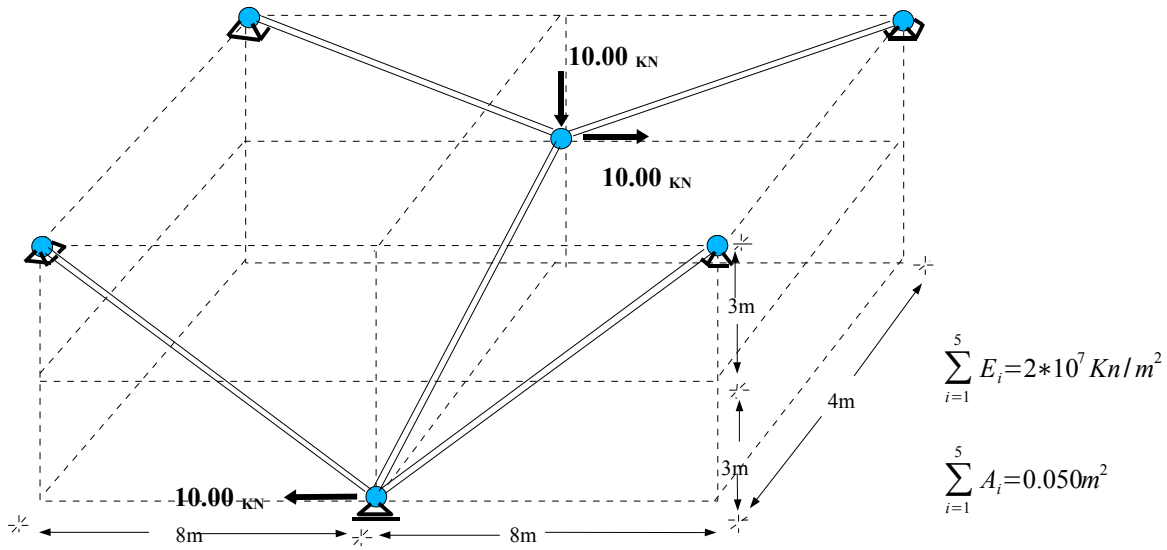
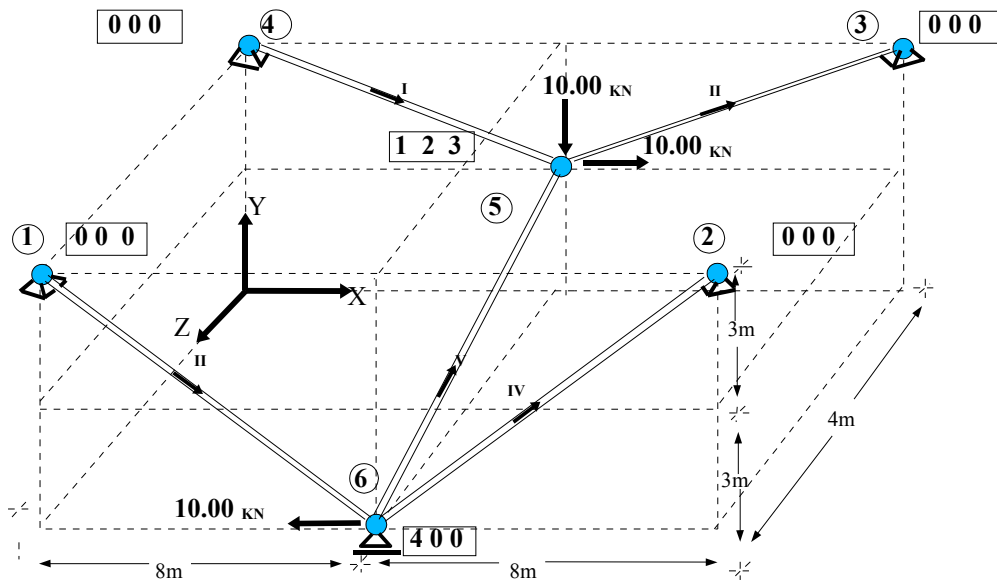


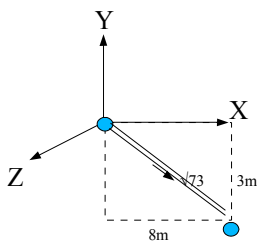
ÖRNEK-4:



SOLUTION-4: All system's elements are node viewpoint, nodes degree of freedom and nodal displacement coefficients.



I. transformation matrix local to global axis



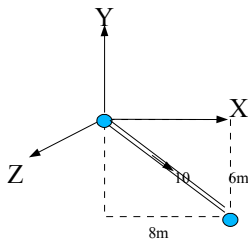
$$[T]_I = \begin{bmatrix} 8/\sqrt{73} & -3/\sqrt{73} & 0.00 & 0.00 & 0.00 & 0.00 \\ 3/\sqrt{73} & 8/\sqrt{73} & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 8/\sqrt{73} & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 8/\sqrt{73} & -3/\sqrt{73} & 0.00 \\ 0.00 & 0.00 & 0.00 & 3/\sqrt{73} & 8/\sqrt{73} & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 8/\sqrt{73} \end{bmatrix}_{6 \times 6}$$

$$l = \cos(\alpha) = 8/\sqrt{73}$$

$$m = \cos(\beta) = -3/\sqrt{73}$$

$$n = \cos(\gamma) = 0.00$$

II. transformation matrix local to global axis



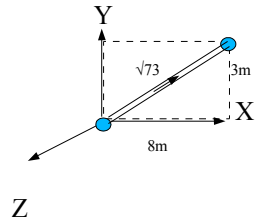
$$[T]_{II} = \begin{bmatrix} 4/5 & -3/5 & 0.00 & 0.00 & 0.00 & 0.00 \\ 3/5 & 4/5 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 4/5 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 4/5 & -3/5 & 0.00 \\ 0.00 & 0.00 & 0.00 & 3/5 & 4/5 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 4/5 \end{bmatrix}_{6 \times 6}$$

$$l = \cos(\alpha) = 4/5$$

$$m = \cos(\beta) = -3/5$$

$$n = \cos(\gamma) = 0.00$$

III. transformation matrix local to global axis



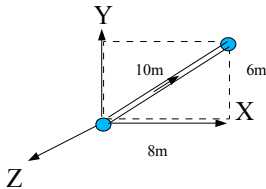
$$[T]_{III} = \begin{bmatrix} 8/\sqrt{73} & 3/\sqrt{73} & 0.00 & 0.00 & 0.00 & 0.00 \\ -3/\sqrt{73} & 8/\sqrt{73} & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 8/\sqrt{73} & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 8/\sqrt{73} & 3/\sqrt{73} & 0.00 \\ 0.00 & 0.00 & 0.00 & -3/\sqrt{73} & 8/\sqrt{73} & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 8/\sqrt{73} \end{bmatrix}_{6 \times 6}$$

$$l = \cos(\alpha) = 8/\sqrt{73}$$

$$m = \cos(\beta) = 3/\sqrt{73}$$

$$n = \cos(\gamma) = 0.00$$

IV. transformation matrix local to global axis



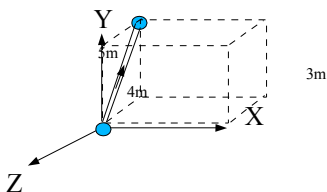
$$[T]_{IV} = \begin{bmatrix} 4/5 & 3/5 & 0.00 & 0.00 & 0.00 & 0.00 \\ -3/5 & 4/5 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 4/5 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 4/5 & 3/5 & 0.00 \\ 0.00 & 0.00 & 0.00 & -3/5 & 4/5 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6}$$

$$l = \cos(\alpha) = 4/5$$

$$m = \cos(\beta) = 3/5$$

$$n = \cos(\gamma) = 0.00$$

V. transformation matrix local to global axis



$$[T]_{V} = \begin{bmatrix} 0.00 & 3/5 & -4/5 & 0.00 & 0.00 & 0.00 \\ -3/5 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 4/5 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 3/5 & -4/5 \\ 0.00 & 0.00 & 0.00 & -3/5 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 4/5 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6}$$

$$l = \cos(\alpha) = 0.00$$

$$m = \cos(\beta) = 3/5$$

I. element global axis stiffness matrix

$$[K]_I = \begin{matrix} & \begin{matrix} 0 & 0 & 0 & 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 64/73 & -24/73 & 0.00 & -64/73 & 24/73 & 0.00 \\ -24/73 & 9/73 & 0.00 & 24/73 & -9/73 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -64/73 & 24/73 & 0.00 & 64/73 & -24/73 & 0.00 \\ 24/73 & -9/73 & 0.00 & -24/73 & 9/73 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6} & \begin{matrix} AE/\sqrt{73} \\ \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} \end{matrix} \end{matrix}$$

II. element global axis stiffness matrix

$$[K]_{II} = \begin{matrix} & \begin{matrix} 0 & 0 & 0 & 4 & 0 & 0 \end{matrix} \\ \begin{bmatrix} 16/25 & -12/25 & 0.00 & -16/25 & 12/25 & 0.00 \\ -12/25 & 9/25 & 0.00 & 12/25 & -9/25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -16/25 & 12/25 & 0.00 & 16/25 & -12/25 & 0.00 \\ 12/25 & -9/25 & 0.00 & -12/25 & 9/25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6} & \begin{matrix} AE/10 \\ \begin{matrix} 0 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \end{matrix} \end{matrix} \end{matrix}$$

III. element global axis stiffness matrix

$$[K]_{III} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 0 & 0 & 0 \end{matrix} \\ \begin{bmatrix} 64/73 & 24/73 & 0.00 & -64/73 & -24/73 & 0.00 \\ 24/73 & 9/73 & 0.00 & -24/73 & -9/73 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -64/73 & -24/73 & 0.00 & 64/73 & 24/73 & 0.00 \\ -24/73 & -9/73 & 0.00 & 24/73 & 9/73 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6} & \begin{matrix} AE/\sqrt{73} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix} \end{matrix} \end{matrix}$$

IV. element global axis stiffness matrix

$$[K]_{IV} = \begin{matrix} & \begin{matrix} 4 & 0 & 0 & 0 & 0 & 0 \end{matrix} \\ \begin{bmatrix} 16/25 & 12/25 & 0.00 & -16/25 & -12/25 & 0.00 \\ 12/25 & 9/25 & 0.00 & -12/25 & -9/25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -16/25 & -12/25 & 0.00 & 16/25 & 12/25 & 0.00 \\ -12/25 & -9/25 & 0.00 & 12/25 & 9/25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6} & \begin{matrix} AE/10 \\ \begin{matrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \end{matrix} \end{matrix}$$

IV. element global axis stiffness matrix

$$[K]_v = \begin{matrix} & \begin{matrix} 4 & 0 & 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 9/25 & -12/25 & 0.00 & -9/25 & 12/25 \\ 0.00 & -12/25 & 16/25 & 0.00 & 12/25 & -16/25 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -9/25 & 12/25 & 0.00 & 9/25 & -12/25 \\ 0.00 & 12/25 & -16/25 & 0.00 & -12/25 & 16/25 \end{matrix} & \begin{matrix} 4 \\ 0 \\ 0 \\ 2 \\ 1 \\ 3 \end{matrix} \end{matrix} \frac{AE}{5}$$

System stiffness matrix

$$[K]_{123} = \begin{bmatrix} 64/73 & -24/73 & 0.00 \\ -24/73 & 9/73 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix} \frac{AE}{\sqrt{73}} + \begin{bmatrix} 64/73 & 24/73 & 0.00 \\ 24/73 & 9/73 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix} \frac{AE}{\sqrt{73}} = \begin{bmatrix} 0.00 & 0.00 & 0.00 \\ 0.00 & 9/25 & -12/25 \\ 0.00 & -12/25 & 16/25 \end{bmatrix} \frac{AE}{5}$$

$$[K]_{123} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0.2052228 & 0.00 & 0.00 \\ 0.00 & 0.100859 & -0.096 \\ 0.00 & -0.096 & 0.128 \end{matrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix} AE$$

$$K_{34} = 0.00 + 0.00 = 0.00$$

$$K_{44} = 16/25 * AE/10 + 16/25 * AE/10 + 0.00 = 0.128 * AE$$

System displacements

$$[K]\{D\} = \{P\}$$

$$\begin{bmatrix} 0.205228 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.10086 & -0.096 & 0.00 \\ 0.00 & -0.096 & 0.128 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.128 \end{bmatrix}_{4 \times 4} AE * \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix}_{4 \times 1} = \begin{Bmatrix} 10.00 \\ -10.00 \\ 0.00 \\ -10.00 \end{Bmatrix}_{4 \times 1}$$

$$D_1 = 48.72752/EA = 48.72752 / (2 * 10^7 * 0.05) = 4.872 \text{ E-5 metre} = +0.048 \text{ mm}$$

$$D_2 = -346.5003/EA = -346.5003 / (2 * 10^7 * 0.05) = -3.465 \text{ E-4 metre} = -0.346 \text{ mm}$$

$$D_3 = -259.8752/EA = -259.8752 / (2 * 10^7 * 0.05) = -2.59875 \text{ E-4 metre} = -0.259 \text{ mm}$$

$$D_4 = -78.125/EA = -78.125 / (2 * 10^7 * 0.05) = -7.8125 \text{ E-5 metre} = -0.078 \text{ mm}$$

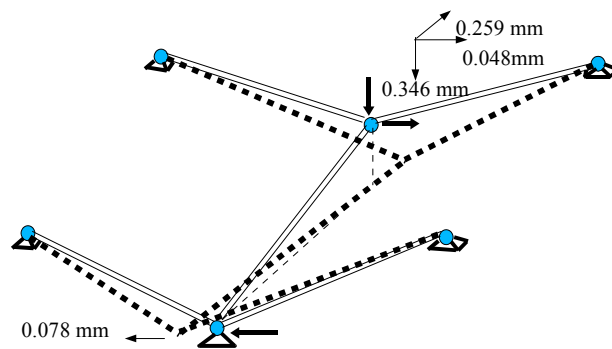
System deformation shape

$$D_1 = +0.048 \text{ mm}$$

$$D_2 = -0.346 \text{ mm}$$

$$D_3 = -0.259 \text{ mm}$$

$$D_4 = -0.078 \text{ mm}$$



I. member's global edge reactions

$$[P]_I = \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 3 \\ 64/73 & -24/73 & 0.00 & -64/73 & 24/73 & 0.00 \\ -24/73 & 9/73 & 0.00 & 24/73 & -9/73 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -64/73 & 24/73 & 0.00 & 64/73 & -24/73 & 0.00 \\ 24/73 & -9/73 & 0.00 & -24/73 & 9/73 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} * \begin{bmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 48.72752 \\ -356.5003 \\ -259.8752 \end{bmatrix}_{6 \times 1} / AE = \begin{bmatrix} -18.330 \\ 6.87409 \\ 0.0000 \\ 18.8330 \\ -6.87490 \\ 0.0000 \end{bmatrix}_{6 \times 1}$$

II. member's global edge reactions

$$[P]_{II} = \begin{bmatrix} 0 & 0 & 0 & 4 & 0 & 0 \\ 16/25 & -12/25 & 0.00 & -16/25 & 12/25 & 0.00 \\ -12/25 & 9/25 & 0.00 & 12/25 & -9/25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -16/25 & 12/25 & 0.00 & 16/25 & -12/25 & 0.00 \\ 12/25 & -9/25 & 0.00 & -12/25 & 9/25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \end{bmatrix} * \begin{bmatrix} 0.00 \\ 0.00 \\ 0.00 \\ -78.125 \\ 0.00 \\ 0.00 \end{bmatrix}_{6 \times 1} / AE = \begin{bmatrix} 5.000 \\ -3.750 \\ 0.000 \\ -5.000 \\ 3.750 \\ 0.000 \end{bmatrix}_{6 \times 1}$$

III. member's global edge reactions

$$[P]_{III} = \begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 64/73 & 24/73 & 0.00 & -64/73 & -24/73 & 0.00 \\ 24/73 & 9/73 & 0.00 & -24/73 & -9/73 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -64/73 & -24/73 & 0.00 & 64/73 & 24/73 & 0.00 \\ -24/73 & -9/73 & 0.00 & 24/73 & 9/73 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} * \begin{bmatrix} 48.72752 \\ -346.5003 \\ -259.8752 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}_{6 \times 1} / AE = \begin{bmatrix} -8.333 \\ -3.1249 \\ 0.000 \\ 8.3330 \\ 3.1249 \\ 0.000 \end{bmatrix}_{6 \times 1}$$

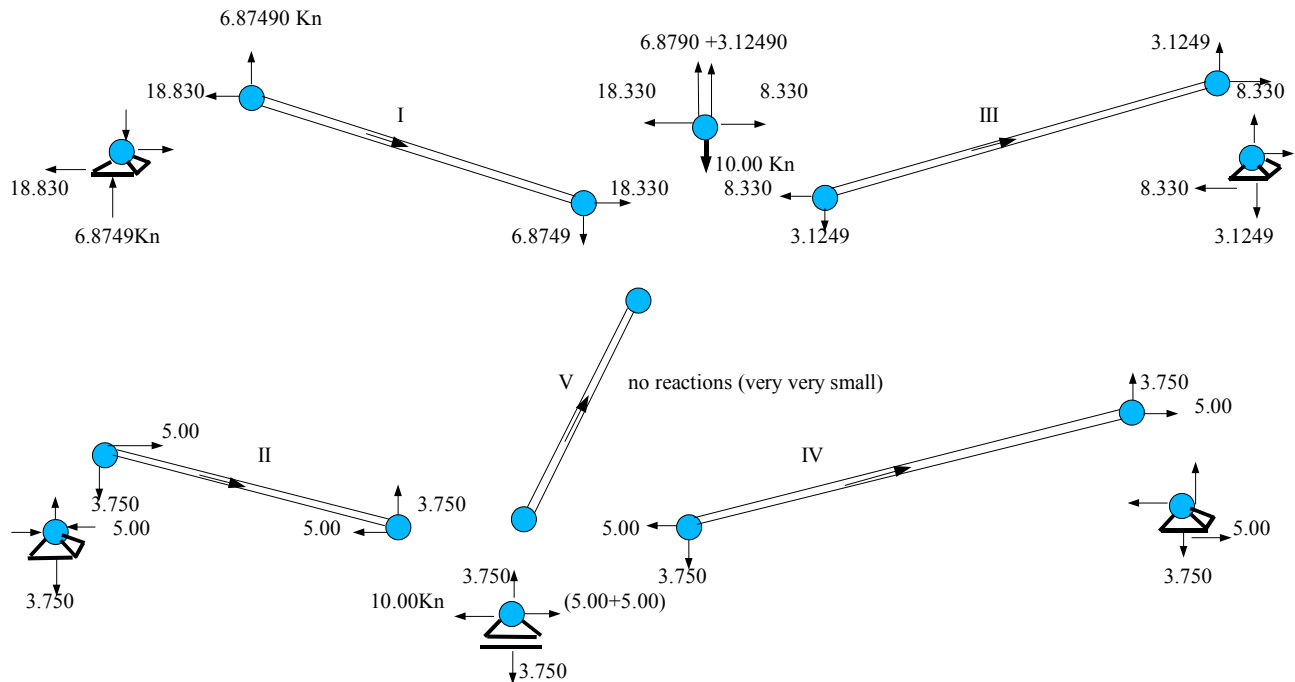
IV. member's global edge reactions

$$[P]_{IV} = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 16/25 & 12/25 & 0.00 & -16/25 & -12/25 & 0.00 \\ 12/25 & 9/25 & 0.00 & -12/25 & -9/25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -16/25 & -12/25 & 0.00 & 16/25 & 12/25 & 0.00 \\ -12/25 & -9/25 & 0.00 & 12/25 & 9/25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6} \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} * \begin{bmatrix} -78.125 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}_{6 \times 1} / AE = \begin{bmatrix} -5.000 \\ -3.750 \\ 0.000 \\ 5.000 \\ 3.750 \\ 0.000 \end{bmatrix}_{6 \times 1}$$

V. member's global edge reactions

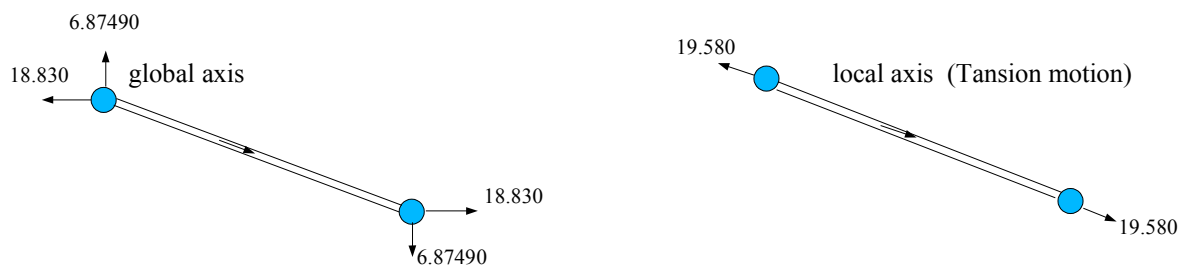
$$[P]_V = \begin{bmatrix} 4 & 0 & 0 & 1 & 2 & 3 \\ 16/25 & 12/25 & 0.00 & -16/25 & -12/25 & 0.00 \\ 12/25 & 9/25 & 0.00 & -12/25 & -9/25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -16/25 & -12/25 & 0.00 & 16/25 & 12/25 & 0.00 \\ -12/25 & -9/25 & 0.00 & 12/25 & 9/25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6} \begin{bmatrix} 4 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} * \begin{bmatrix} -78.125 \\ 0.00 \\ 0.00 \\ 48.7275 \\ -346.5003 \\ -259.8752 \end{bmatrix}_{6 \times 1} / AE = \begin{bmatrix} 0.000 \\ 2.40 * 10^{-6} \\ -3.2 * 10^{-6} \\ 0.000 \\ -2.40 * 10^{-6} \\ 3.20 * 10^{-6} \end{bmatrix}_{6 \times 1}$$

Node equilibrium method



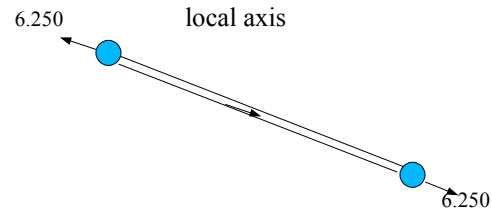
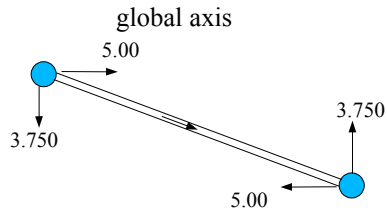
I. member local axis node reactions

$$[P]_L = \begin{bmatrix} 8/\sqrt{73} & -3/\sqrt{73} & 0.00 & 0.00 & 0.00 & 0.00 \\ 3/\sqrt{73} & 8/\sqrt{73} & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 8/\sqrt{73} & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 8/\sqrt{73} & -3/\sqrt{73} & 0.00 \\ 0.00 & 0.00 & 0.00 & 3/\sqrt{73} & 8/\sqrt{73} & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 8/\sqrt{73} \end{bmatrix}_{6 \times 6} * \begin{Bmatrix} -18.330 \\ 6.87409 \\ 0.0000 \\ 18.8330 \\ -6.87490 \\ 0.0000 \end{Bmatrix}_{6 \times 1} = \begin{Bmatrix} -19.580 \\ 0.000 \\ 0.000 \\ 19.580 \\ 0.000 \\ 0.000 \end{Bmatrix}_{6 \times 1}$$



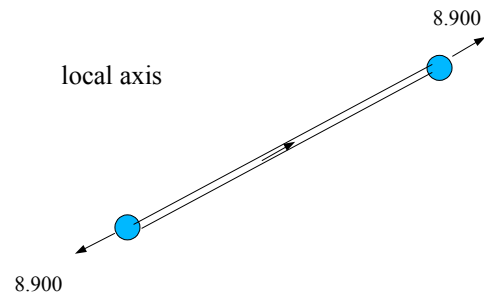
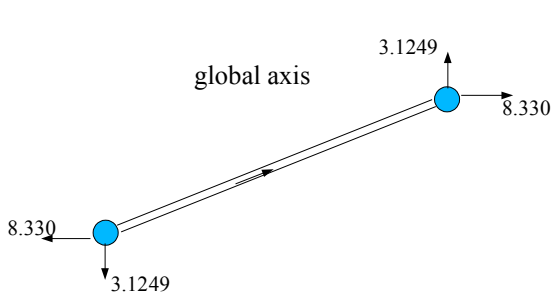
II. member local axis node reactions

$$[P]_L = \begin{bmatrix} 4/5 & -3/5 & 0.00 & 0.00 & 0.00 & 0.00 \\ 3/5 & 4/5 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 4/5 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 4/5 & -3/5 & 0.00 \\ 0.00 & 0.00 & 0.00 & 3/5 & 4/5 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 4/5 \end{bmatrix}_{6 \times 6} * \begin{Bmatrix} 5.000 \\ -3.750 \\ 0.000 \\ -5.000 \\ 3.750 \\ 0.000 \end{Bmatrix}_{6 \times 1} = \begin{Bmatrix} 6.250 \\ 0.000 \\ 0.000 \\ -6.250 \\ 0.000 \\ 0.000 \end{Bmatrix}_{6 \times 1}$$



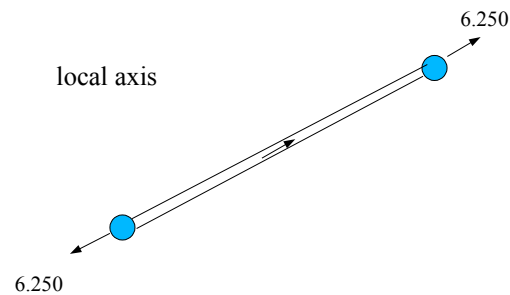
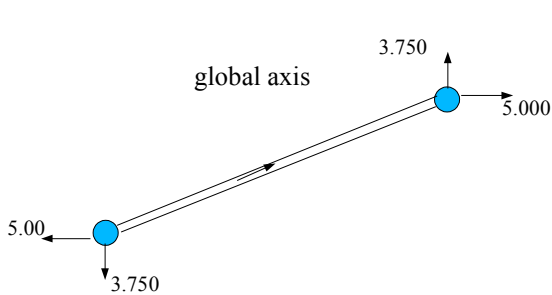
III. member local axis node reactions

$$[P]_L = \begin{bmatrix} 8/\sqrt{73} & 3/\sqrt{73} & 0.00 & 0.00 & 0.00 & 0.00 \\ -3/\sqrt{73} & 8/\sqrt{73} & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 8/\sqrt{73} & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 8/\sqrt{73} & 3/\sqrt{73} & 0.00 \\ 0.00 & 0.00 & 0.00 & -3/\sqrt{73} & 8/\sqrt{73} & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 8/\sqrt{73} \end{bmatrix}_{6 \times 6} * \begin{Bmatrix} -8.3330 \\ -3.12490 \\ 0.0000 \\ 8.3330 \\ 3.12490 \\ 0.0000 \end{Bmatrix}_{6 \times 1} = \begin{Bmatrix} -8.900 \\ 0.000 \\ 0.000 \\ 8.900 \\ 0.000 \\ 0.000 \end{Bmatrix}_{6 \times 1}$$



IV. member local axis node reactions

$$[P]_L = \begin{bmatrix} 4/5 & 3/5 & 0.00 & 0.00 & 0.00 & 0.00 \\ -3/5 & 4/5 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 4/5 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 4/5 & 3/5 & 0.00 \\ 0.00 & 0.00 & 0.00 & -3/5 & 4/5 & 0.00 \\ 0.00 & 0.00 & 0.00 & 4/5 & 0.00 & 0.00 \end{bmatrix}_{6 \times 6} * \begin{Bmatrix} -5.000 \\ -3.750 \\ 0.000 \\ 5.000 \\ 3.750 \\ 0.000 \end{Bmatrix}_{6 \times 1} = \begin{Bmatrix} -6.250 \\ 0.000 \\ 0.000 \\ 6.250 \\ 0.000 \\ 0.000 \end{Bmatrix}_{6 \times 1}$$



Element No	Force (KN)	Type
I	19.5799	Tension (+)
II	6.25000	Pressure (-)
III	8.89934	Tension (+)
IV	6.25000	Tension (+)
V	0.00000	---