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Arbitrage strategy of virtual power plants in energy, spinning reserve and reactive power markets

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Abstract: Virtual power plant (VPP) concept was developed to integrate distributed energy resources (DERs) into the grid in order that they are seen as a single power plant by the market and power system operator. Therefore, VPPs are faced with optimal bidding, and identifying arbitrage opportunities in a market environment. In this study, the authors present an arbitrage strategy for VPPs by participating in energy and ancillary service (i.e. spinning reserve and reactive power services) markets. On the basis of a security-constrained price-based unit commitment, their proposed model maximises VPP's profit (revenue minus costs) considering arbitrage opportunities. The supply–demand balancing, transmission network topology and security constraints are considered to ensure reliable operation of VPP. The mathematical model is a mixed-integer non-linear optimisation problem with inter-temporal constraints, and solved by mixed-integer non-linear programming. The result is a single optimal bidding profile and a schedule for managing active and reactive power under participating in the markets. These profile and schedule consider the DERs and network constraints simultaneously, and explore arbitrage opportunities of VPP. Results pertaining to an illustrative example and a case study are discussed.

Nomenclature

Sets and indices

$n,\ m$	index for buses
t	index for hours
w	set of end consumers
z	set of ILs
v	set of DGs
u	set of ESSs
g	set of CBs
s	set of number of CB steps

Parameters

$\rho_t^{\text{active}},\ \rho_t^{\text{reactive}}$	VPP's active and reactive retail rates
$P_{wt}^{\text{D}},\ Q_{wt}^{\text{D}}$	active and reactive demands
$P_z^{\text{ILmax}},\ P_z^{\text{ILmin}}$	upper and lower limits on curtailment of IL active power
$P_v^{\text{dgmax}},\ P_v^{\text{dgmin}}$	upper and lower limits on active power generation of a DG
$Q_v^{\text{dgmax}},\ Q_v^{\text{dgmin}}$	upper and lower limits on reactive power generation of a DG
S_v^{dgmax}	DG maximum apparent power
H_v^{ILmax}	number of permitted hours for IL curtailment
P_u^{strmax}	installed capacity of ESS
$\text{DOD}_u^{\text{str}}$	ESS DOD
P_u^{initial}	ESS initial state of charge
R_u^{str}	maximum charge/discharge rate of ESS
No_g^{CBmax}	number of CB steps
$V_n^{\max},\ V_n^{\min}$	maximum/minimum voltage magnitude
F_{nm}^{vpp}	apparent power capacity of the transmission lines

Variables

$P_{vt}^{\text{dg}},\ R_{vtc}^{\text{dg}},\ Q_{vt}^{\text{dg}}$	DG generation for energy, spinning reserve and reactive power markets, respectively
$P_{zt}^{\text{IL}},\ R_{ztc}^{\text{IL}}$	IL curtailment for energy and spinning reserve markets, respectively
V_{ntc}	voltage amplitude
$\delta_{ntc}^{\text{vpp}}$	voltage angle
P_{nmc}^{vpp}	active power flow of transmission line
Q_{nmc}^{vpp}	reactive power flow of transmission line

Binary variables

I_{vt}^{dg}	commitment status of a DG
$I_{vt}^{\text{dg-React}}$	binary variable denoting provision of reactive power by a DG
$J_{vt}^{\text{dg}},\ K_{vt}^{\text{dg}}$	start-up and shutdown decision for a DG
I_{zt}^{int}	curtailment status of an IL
I_{gst}^{CB}	switching status of CB step
$J_{gst}^{\text{CB}},\ K_{gst}^{\text{CB}}$	switch on and off decisions for a CB step

1 Introduction

Recently, much attention has been given to the use of distributed energy resources (DERs), including distributed generation (DG), energy storage system (ESS) and demand response (DR) for increasing the energy efficiency of power systems [1]. By increasing the penetration of DERs, two major problems were found: (a) these elements were not visible for the independent system operator (ISO) to operate them efficiently and optimally [2–5] and (b) they could not participate in energy and ancillary service markets to earn profits [4–8]. Notable that services supplied to the ISOs by the market participants to maintain the power system reliability and security are called 'ancillary services' [9, 10]. To overcome these problems, innovative concepts such as virtual power plant (VPP) have been proposed [1, 2]. By integration of DERs, VPPs make them visible to the ISO in order to provide presence of DERs in the markets [8, 11].

The objective of VPP is profit maximisation by identifying arbitrage opportunities and optimal bidding in the markets.

Arbitrage is the process of earning riskless profits by taking advantage of differential pricing for the same physical asset or security. As a widely applied investment tactic, arbitrage typically entails the sale of a security at a relatively high price and the simultaneous purchase of the same security (or its functional equivalent) at a relatively low price [12]. Arbitrage also makes references to any activity that attempts to buy a relatively under-priced commodity and to sell a similar and relatively over-priced commodity for profit [13]. In this paper, two definitions of arbitrage are considered in order to be able to provide earning more money.

Though a long-term planning study has been done to find the optimum selection of energy storage technology based on net present value by considering the arbitrage opportunity of energy storage in [14], our paper is concentrated on short-term planning by considering all kinds of arbitrage opportunities of VPP via optimal bidding in a day-ahead energy and ancillary service markets. The VPP bidding can be categorised into two groups: equilibrium and non-equilibrium models [15]. Strategic bidding of VPP via Nash-supply function equilibrium (SFE) equilibrium model in an energy market has been proposed in [16, 17]. Owing to some computational problems, equilibrium models are not applicable to a large system with many market participants. Therefore, a non-equilibrium model based on a security constraints price-based unit commitment (SCPBUC) for the bidding strategy of a VPP has been presented in [18, 19]. Consequently, the provision of energy and spinning reserve has been considered in these papers. A probabilistic price-based unit commitment (PBUC) approach is employed for optimal bidding of a VPP in a day-ahead energy market in [20]. In [21, 22], the bidding problem is cast as two-stage programming models which maximise the VPP expected profit. Mixed-integer non-linear programming (MINLP) and mixed-integer linear programming (MILP) are used to solve these proposed models, respectively. For the sake of comparison, the features of the proposed model in our paper and other relevant works are summarised in Table 1.

To earn more profit, VPP should identify arbitrage opportunities and optimal bidding strategies in energy, spinning reserve and reactive power markets. Thus, a non-equilibrium model based on the SCPBUC has been extended in this paper. In SCPBUC model, energy and ancillary services can be optimised simultaneously, so the results provide a portfolio of energy and ancillary services bids. These results can be used for exploring arbitrage opportunities between energy and ancillary services. As far as we know, such a practical and profitable model of VPP, in which considers not only the simultaneous management of active and reactive powers, but also the arbitrage strategy of VPP, has not been presented. Besides and more importantly, the previous works focus on active power management frameworks, whereas our proposed model simultaneously considers both active and reactive power managements for a joint market of energy and spinning reserve services, coupled with reactive power market. Therefore, the contributions of this paper can be briefly summarised as follows:

(i) Exploring arbitrage strategy of VPP in energy, spinning reserve and reactive power markets.
(ii) Proposing a model which allows VPP to manage active and reactive powers in a joint market of energy and spinning reserve services, coupled with reactive power market, and to consider the reactive demand charges of VPP when the power factor deviates from the permissible range.
(iii) Exchanging energy and ancillary services to the upstream network via different grid supply points (GSPs) to decrease energy losses.

This paper has been organised as follows: Section 2 presents some challenges on VPP concept. The mathematical model is described in Section 3. Section 4 deals with the test system used in this paper. It also includes a brief summary of the simulation used to obtain the results, numerical results and some observations and discussion. Finally, the conclusions of this paper are summarised in Section 5.

Table 1 Features of the proposed model in this paper and other relevant works

References	Model	Management of active and reactive power	Exploration of arbitrage strategy	Consideration of reactive demand charges	Consideration of network aspects of aggregation	Different GSPs	VPP components				Types of markets	Approach
							DG	ESS	DR	CB		
[16]	Nash-SFE equilibrium	×	×	×	×	–	√	×	×	×	energy market	iterative algorithm
[17]	Nash-SFE equilibrium	×	×	×	×	–	√	×	√	×	energy market	iterative algorithm
[18, 19]	SCPBUC	×	×	×	√	×	√	√	√	×	energy and spinning reserve markets	genetic algorithm
[20]	PBUC	×	×	×	√	√	√	×	√	×	energy market	MINLP
[21]	two-stage programming	×	×	×	√	√	√	×	√	×	energy market	MINLP
[22]	two-stage programming	×	×	×	×	–	√	√	×	×	energy and balancing markets	MILP
this paper	SCPBUC	√	√	√	√	√	√	√	√	√	energy, spinning reserve and reactive power markets	MINLP

2 VPP challenges

2.1 VPP's definition

There is no consensus regarding the definition of VPP [2]. The universe of VPP is subdivided into three sections: DR-based VPP, supply-side VPP and mixed-asset VPP [23]. A mixed-asset VPP, which is the target of VPP and brings DERs (i.e. DG, ESS and DR programmes), has been investigated in this paper.

There are several ways to aggregate DERs by VPPs, in order that the aggregation approaches of DERs have been classified in [3, 8]. There are two types of aggregation: commercial VPP and technical VPP (TVPP). In this paper, we consider TVPP regarding the provision of energy and ancillary services at multiple GSPs. It includes the real-time impact of the local network on DER aggregated profile, and represents the cost and operating characteristics of the portfolio.

2.2 Energy and ancillary service providers

To maintain the frequency and the voltage of the power system within the allowable limits, the ISO has to procure sufficient spinning reserve as frequency control ancillary service and sufficient reactive power, respectively [9, 24].

DERs including DGs, reactive power compensators [i.e. capacitor bank (CB)], ESSs and interruptible loads (ILs) can provide energy and ancillary services [25–28]. In the following, capabilities of the DERs to providing energy and ancillary services are described.

With respect to the grid-coupling converters capabilities, DGs can provide energy and ancillary services. There are three kinds of limitations on DG production: active power generation limit, reactive power generation limit and maximum apparent power limit [26]. According to the capability curve of DGs, they can simultaneously provide energy, spinning reserve and reactive power services. We consider that ESSs can only provide energy service. We need to make a decision on scheduling charge and discharge of the ESS on a time frame basis. Hence, they have been modelled by modifying [18]. CBs have a limit on daily switching [29]. With providing reactive power service, CBs consist of discrete capacitor steps with a maximum allowable daily switching operation number (MADSON).

In case of ILs, the upper limit of curtailing, load power factor and permitted hours for curtailing are determined. As a result, ILs can simultaneously provide energy and spinning reserve service. Moreover, VPP signs a contract with non-interruptible consumers to supply active and reactive demands.

2.3 VPP's arbitrage strategy

First, energy arbitrage refers to purchasing energy when electricity prices are low and selling energy when electricity prices are high [13]. This strategy of VPP is achievable by ESSs. Second, if there are any bilateral contracts, a VPP may satisfy them either by purchases from the market or by local generation. The arbitrage between local generation and purchases from the market would be more profitable than purchases from the market alone [30]. Therefore, VPP can supply active/reactive load by local generation or purchases from energy/reactive power market. Third, the arbitrage across energy and spinning reserve commodities may provide a higher profit for a VPP than selling energy alone as a commodity [31].

2.4 Market model

To organise complexities, different ancillary services can be cleared sequentially or simultaneously in the markets, though they are produced simultaneously [9, 13, 24, 32]. In our paper, we consider a hybrid model of clearing. The energy and spinning reserve markets are cleared simultaneously, and then the reactive power market is cleared sequentially. This kind of clearing can be called 'a joint market of energy and spinning reserve service, coupled with reactive power market'. Therefore, we assumed that VPP participated in a joint market of energy and spinning reserve service, coupled with reactive power market. A double-sided auction mechanism is considered for ISO's day-ahead market. All of market participants submit their bids and offers with price and megawatt (MW) pairs for energy and spinning reserve markets, as well as with price and mega volt ampere reactive (MVar) pair for reactive power market. The VPP's optimal bidding quantity (i.e. MW or MVar) has been determined using the proposed model based on the forecasted prices. In other words, if VPP bids a little lower than the forecasted price of the markets, it will win the amount of optimal quantity (i.e. MW or MVar) in the market. Hence, both of price and quantity have been submitted to the markets in our model.

3 Model description

3.1 Optimal bidding of VPP via SCPBUC considering arbitrage opportunities

Since this paper is concerned on considering arbitrage opportunities of VPP by optimal bidding in energy, spinning reserve and reactive power markets, the SCPBUC has been extended to design the optimal bidding of VPP. The objective function of the above problem is to maximise the expected value of the profit. The settlement process of the markets can be in the forms of: uniform or pay-as-bid. Thus, the pay-as-bid mechanism is used in this paper.

Whether the spinning reserve is called onto produce or not, VPP must operate its DERs reliably and securely. In this condition, steady-state security constraints of VPP, supply–demand balancing constraints and some DERs constraints, which are concerned by not calling on spinning reserve, must be checked again. This problem has been modelled as two contingencies ($c \in \{0,\ 1\}$) with equal probability, where $c = \{1\}$ and $c = \{0\}$, shown the normal condition that the spinning reserve is called on, and is not called on by the ISO, respectively. This way of modelling is a conventional approach in which applicable in power system [33].

3.2 Assumptions

The information required by VPP to optimise its arbitrage strategy is assumed as follows:

(i) VPP loads and market prices can be forecasted based on historical data. Noteworthily that the cost of reactive power generation is much lower than that of active power because it does not involve fuel costs [10]; therefore, the prices of energy and spinning reserve markets are more expensive than that of reactive power market.

(ii) Generally, the spinning reserve market is settled based on the bids for capacity. However in some markets, the winner participants are additionally paid for the amount of energy which has been called onto produce [18]. In addition, the loss of opportunity caused by reactive power generation may be additionally paid to the winners in some reactive power market [10]. For the sake of simplicity, it is assumed that VPP does not consider the part of profit in which related to the amount of energy which will be called onto produce in spinning reserve market, and also to the amount of opportunity loss in reactive power market. Therefore, an SCPBUC has been proposed based on the forecasted price of spinning reserve market concerned with the capacity, and the forecasted price of reactive power market without considering the amount of opportunity loss. Thus, VPP will maximise its minimum expected profit.

(iii) If the absolute value of VPP power factor ($|\mathrm{Cos}\ \varphi_{jtc}^{\mathrm{VPP}}|$) is less than $\mathrm{Cos}\ \varphi_j^{\min} = 0.95$, then VPP will be penalised an additional fee. The power factor penalty is modelled as (1) (see equation (1) at the bottom of the next page)

$$\mathrm{Cos}\ \varphi_{jtc}^{\mathrm{VPP}} = \frac{P_{jt}^{\mathrm{Energy}} + R_{jtc}^{\mathrm{Spinning\ Reserve}}}{\sqrt{\left(P_{jt}^{\mathrm{Energy}} + R_{jtc}^{\mathrm{Spinning\ Reserve}}\right)^2 + Q_{jt}^{\mathrm{Reactive}^2}}} \tag{2}$$

where α_{jt}^{PF} is the penalty coefficient; and P_{jt}^{Energy}, Q_{jt}^{Reactive}, and $R_{jtc}^{\text{Spinning Reserve}}$ are, respectively, bidding quantity to energy, reactive power and spinning reserve markets at GSP j and time t.

(iv) The retail rates are specified by the VPP's bilateral contract with the end consumers.

(v) The DG units are assumed dispatchable, and the production cost of each DG is considered as (3)

$$C_{vt}^{\text{dg}}\left(P_{vt}^{\text{dg}},\, Q_{vt}^{\text{dg}}\right) = a_{vt}^{\text{dg}}.\left(P_{vt}^{\text{dg}}\right) + b_{vt}^{\text{dg}} + \alpha_{vt}^{\text{dg}}.\left|Q_{vt}^{\text{dg}}\right| + \beta_{vt}^{\text{dg}} \tag{3}$$

where a_{vt}^{dg} is the variable cost of active power generation; b_{vt}^{dg} is a fixed cost, which is paid caused by availability of DG to provide energy and spinning reserves; α_{vt}^{dg} is the reactive power injection or absorption cost; and β_{vt}^{dg} is an availability payment component (for that portion of a DG capital cost that is relevant to reactive power production). Finally, the start-up ($\text{SC}_{vt}^{\text{dg}}$) and shutdown ($\text{SHC}_{vt}^{\text{dg}}$) costs are paid to the DG.

(vi) The cost of load curtailment is modelled as (4) for ILs

$$C_{zt}^{\text{IL}}\left(P_{zt}^{\text{IL}}\right) = a_{zt}^{\text{IL}}.\left(P_{zt}^{\text{IL}}\right) + b_{zt}^{\text{IL}} \tag{4}$$

where a_{zt}^{IL} is the variable cost of unserved active demand and b_{zt}^{IL} is a fixed cost, which is hourly paid to the ILs to prevent multiple switching.

(vii) The cost of ESS is modelled as (5)

$$C_{ut}^{\text{str}}\left(P_{ut}^{\text{str}}\right) = \alpha_{ut}^{\text{str}}\left|P_{ut}^{\text{str}}\right| + \beta_{ut}^{\text{str}} \tag{5}$$

The operational cost of ESS is generally involved with maintenance costs, and based on [17] it is assumed to be a linear function of the absolute of its charged or discharged capacity (P_{ut}^{str}) at each hour.

(viii) The cost of CB is modelled as (6)

$$C_{gst}^{\text{CB}}(\bar{Q}_{gst}^{\text{CB}}) = \alpha_{gst}^{\text{CB}}.\bar{Q}_{gst}^{\text{CB}} \tag{6}$$

where $\bar{Q}_{gst}^{\text{CB}}$ is the discrete value of each CB step and α_{gst}^{CB} is a fixed cost which is paid to each step of CB.

3.3 Problem formulation

3.3.1 Objective function: To calculate the minimum expected profit of VPP, spinning reserve has been considered as 'called on' by the ISO

$$\text{Maximise Profit} = \sum_t \left\{\text{Revenue}_t - \text{Cost}_t\right\} \tag{7}$$

(see (8))

$$\text{Cost}_t = \sum_v C_{vt}^{\text{dg}}\left(P_{vt}^{\text{dg}} + R_{vt(c=\{1\})}^{\text{dg}},\, Q_{vt}^{\text{dg}}\right) + \sum_z C_{zt}^{\text{IL}}\left(P_{zt}^{\text{IL}} + R_{zt(c=\{1\})}^{\text{IL}}\right) + \sum_u C_{ut}^{\text{str}}\left(P_{ut}^{\text{str}}\right) + \sum_{gs} C_{gst}^{\text{CB}}(\bar{Q}_{gst}^{\text{CB}}) + \sum_{(j\in\Psi_n^{\text{vpp}})} \text{Penalty}_{jt}^{\text{PF}}(Q_{jt}^{\text{Reactive}}) \tag{9}$$

As mentioned before, the provision of energy and ancillary services is possible in multiple GSPs. Note that $j \in \Psi_n^{\text{vpp}}$ identifies the GSPj located at the nth bus of VPP. $\lambda_{jt}^{\text{Energy}}$, $\lambda_{jt}^{\text{Spinning Reserve}}$ and $\lambda_{jt}^{\text{Reactive}}$ are, respectively, the forecasted prices of energy, reactive power and spinning reserve markets at GSP j and time t. In the following, the constraints are given.

3.3.2 Constraints: *DG constraints:* Equations (10)–(12) enforce capability curve limits, which are described in Section 2.3. Equations (13) and (14) enforce rates of ramp up and ramp down, respectively. The constraints in (15) enforce the ramping capability for spinning reserve. Binary constraints in (16) enforce the start-up and shutdown of DGs. Finally, (17) and (18) enforce the minimum up and minimum down times

$$P_v^{\text{dgmin}} \cdot I_{vt}^{\text{dg}} \le P_{vt}^{\text{dg}} + R_{vtc}^{\text{dg}} \le P_v^{\text{dgmax}} \cdot I_{vt}^{\text{dg}}\;,\; \forall v,\, \forall t,\, \forall c \tag{10}$$

$$Q_v^{\text{dgmin}} \cdot I_{vt}^{\text{dg}-\text{React}} \le Q_{vt}^{\text{dg}} \le Q_v^{\text{dgmax}} \cdot I_{vt}^{\text{dg}-\text{React}},\; \forall v,\, \forall t \tag{11}$$

$$\sqrt{\left(Q_{vt}^{\text{dg}}\right)^2 + \left(P_{vt}^{\text{dg}} + R_{vtc}^{\text{dg}}\right)^2} \le S_v^{\text{dgmax}}\;,\; \forall v,\, \forall t,\, \forall c \tag{12}$$

$$\left(P_{v(t+1)}^{\text{dg}}\right) - \left(P_{vt}^{\text{dg}}\right) \ge RU_v^{\text{dg}}\;,\; \forall v,\, \forall t \tag{13}$$

$$\left(P_{vt}^{\text{dg}}\right) - \left(P_{v(t+1)}^{\text{dg}}\right) \ge RD_v^{\text{dg}}\;,\; \forall v,\, \forall t \tag{14}$$

$$R_{vtc}^{\text{dg}} \le \min\left\{10.\,\text{MSR}_v^{\text{dg}},\, P_v^{\text{dgmax}} \cdot I_{vt}^{\text{dg}} - P_{vt}^{\text{dg}}\right\}\;,\; \forall v,\, \forall t,\, \forall c \tag{15}$$

$$\left\{\begin{array}{c} I_{vt}^{\text{dg}} - I_{v(t-1)}^{\text{dg}} \le J_{vt}^{\text{dg}} \\ I_{v(t-1)}^{\text{dg}} - I_{vt}^{\text{dg}} \le K_{vt}^{\text{dg}} \\ I_{vt}^{\text{dg}} - I_{v(t-1)}^{\text{dg}} \le J_{vt}^{\text{dg}} - K_{vt}^{\text{dg}} \end{array}\right\},\; \forall v,\, \forall t \tag{16}$$

$$\sum_{l=1}^{\text{MUT}_v^{\text{dg}}} I_{v(t+l)}^{\text{dg}} - 1 \ge \text{MUT}_v^{\text{dg}},\; \forall J_{vt}^{\text{dg}} = 1\;,\; \forall v,\, \forall t \tag{17}$$

$$\sum_{l=1}^{\text{MDT}_v^{\text{dg}}} 1 - I_{v(t+l)}^{\text{dg}} \ge \text{MDT}_v^{\text{dg}},\;\; \forall K_{vt}^{\text{dg}} = 1\;,\; \forall v,\, \forall t \tag{18}$$

IL constraints: Equation in (19) enforces active load curtailment limits as energy and spinning reserve. Constraint in (20) enforces the reactive load curtailment in relation with power factor

$$P_z^{\text{ILmin}} \times I_{zt}^{\text{IL}} \le P_{zt}^{\text{IL}} + R_{ztc}^{\text{IL}} \le P_z^{\text{ILmax}} \times I_{zt}^{\text{IL}}\;,\; \forall z,\, \forall t,\, \forall c \tag{19}$$

$$Q_{zt}^{\text{IL}} = \left(P_{zt}^{\text{IL}}\right) \cdot \sqrt{\frac{1}{\cos\varphi_z^{\text{IL}}} - 1}\;,\; \forall z,\, \forall t \tag{20}$$

$$\sum_t I_{zt}^{\text{IL}} \le H_z^{\text{ILmax}}\;,\; \forall z \tag{21}$$

ESS constraints: Equation in (22) enforces maximum capacity storage limit. The constraint in (23) enforces depth of discharge

$$\text{Penalty}_{jt}^{\text{PF}}\left(Q_{jt}^{\text{Reactive}}\right) = \left\{\begin{array}{l} \alpha_{jt}^{\text{PF}}.Q_{jt}^{\text{Reactive}} \text{ if } \left|\text{Cos }\varphi_{jtc}^{\text{VPP}}\right| \le \text{Cos }\varphi_j^{\min} \\ 0 \text{ if } \left|\text{Cos }\varphi_{jtc}^{\text{VPP}}\right| \ge \text{Cos }\varphi_j^{\min} \end{array}\right.,\; \forall j \in \Psi_n^{\text{vpp}},\, \forall c,\, \forall t \tag{1}$$

$$\text{Revenue}_t = \sum_{(j\in\Psi_n^{\text{vpp}})} \lambda_{jt}^{\text{Energy}} \times P_{jt}^{\text{Energy}} + \sum_{(j\in\Psi_n^{\text{vpp}})} \lambda_{jt}^{\text{Spinning Reserve}} \times R_{jt(c=\{1\})}^{\text{Spinning Reserve}} + \sum_{(j\in\Psi_n^{\text{vpp}})} \lambda_{jt}^{\text{Reactive}} \times Q_{jt}^{\text{Reactive}} + \sum_w \rho_t^{\text{active}} \times P_{wt}^D + \sum_w \rho_t^{\text{reactive}} \times Q_{wt}^D + \sum_z \rho_t^{\text{active}} \times \left(P_z^{\text{ILmax}} - P_{zt}^{\text{IL}}\right) + \sum_z \rho_t^{\text{reactive}} \times \left(Q_z^{\text{ILmax}} - Q_{zt}^{\text{IL}}\right) \tag{8}$$

(DOD). Constraint (24) implements the initial state of charge. The constraint in (25) enforces ramp rate of charge/discharge

$$\sum_{k=1}^{t} P_{uk}^{\text{str}} \le P_{u}^{\text{strmax}}\ ,\ \forall u,\ \forall t \tag{22}$$

$$\sum_{k=1}^{t} P_{uk}^{\text{str}} \ge \text{DOD}_{u}^{\text{str}}\ ,\ \forall u,\ \forall t \tag{23}$$

$$P_{u(t=\{1\})}^{\text{str}} = P_{u}^{\text{initial}}\ ,\ \forall u,\ \forall t \tag{24}$$

$$\left|P_{ut}^{\text{str}}\right| \le R_{u}^{\text{str}}\ ,\ \forall u\ ,\ \forall t \tag{25}$$

CB constraints: Equation in (26) enforces number of CB steps. Binary constraints in (27) enforce the start-up and shutdown of the CB steps' switching. Constraint in (28) enforces MADSON of CB steps

$$\sum_{s} I_{gst}^{\text{CB}} \le No_{g}^{\text{CBmax}}\ ,\ \forall g,\ \forall t \tag{26}$$

$$\left\{\begin{array}{c} I_{gst}^{\text{CB}} - I_{gs(t-1)}^{\text{CB}} \le J_{gst}^{\text{CB}} \\ I_{gs(t-1)}^{\text{CB}} - I_{gst}^{\text{CB}} \le K_{ust}^{\text{CB}} \\ I_{gst}^{\text{CB}} - I_{gs(t-1)}^{\text{CB}} \le J_{gst}^{\text{CB}} - K_{gst}^{\text{CB}} \end{array}\right\}\ ,\ \forall g\ ,\ \forall s\ ,\ \forall t \tag{27}$$

$$\sum_{t}\left(J_{gst}^{\text{CB}} + K_{gst}^{\text{CB}}\right) \le \text{MADSON}_{gs}\ ,\ \forall g\ ,\ \forall s \tag{28}$$

Supply–demand balancing constraints: If spinning reserve service called onto produce or not, we see (29) and (30). The first one is related to the active power and the second one is related to the reactive power. Note that $v \in \Psi_n^{\text{vpp}}$ identifies the vth DG located at the nth bus of VPP. Furthermore, $m \in \Phi_n^{\text{vpp}}$ identifies the buses m connected to the nth bus of VPP

$$\left\{\begin{array}{l} \sum_{v\in\Psi_n^{\text{vpp}}}\left(P_{vt}^{\text{dg}}+R_{vtc}^{\text{dg}}\right)-\sum_{j\in\Psi_n^{\text{vpp}}}\left(P_{jt}^{\text{Energy}}+R_{jtc}^{\text{SpinningReserve}}\right) \\ \quad -\sum_{u\in\Psi_n^{\text{vpp}}}\eta_u^{\text{str}}.P_{ut}^{\text{str}}-\sum_{w\in\Psi_n^{\text{vpp}}}P_{wt}^{D}+\sum_{z\in\Psi_n^{\text{vpp}}}\left(P_{zt}^{\text{IL}}+R_{ztc}^{\text{IL}}\right) \qquad \forall n, \forall c, \forall t \\ \quad -\sum_{m\in\Phi_n^{\text{vpp}}}\left|V_{ntc}\right|\cdot\left|V_{mtc}\right|\cdot\left|Y_{nm}\right|\cdot\cos\left(\theta_{nm}^{\text{vpp}}-\delta_{ntc}^{\text{vpp}}+\delta_{mtc}^{\text{vpp}}\right)=0, \end{array}\right. \tag{29}$$

$$\left\{\begin{array}{l} \sum_{v\in\Psi_n^{\text{vpp}}}\left(Q_{vt}^{\text{dg}}\right)-\sum_{j\in\Psi^{\text{GSP}}}Q_{jt}^{\text{Reactive}}-\sum_{w\in\Psi_n^{\text{vpp}}}Q_{wt}^{D}+\sum_{z\in\Psi_n^{\text{vpp}}}\left(Q_{zt}^{\text{IL}}\right) \qquad \forall n, \forall c, \forall t \\ \quad +\sum_{m\in\Phi_n^{\text{vpp}}}\left|V_{ntc}\right|\cdot\left|V_{mtc}\right|\cdot\left|Y_{nm}\right|\cdot\sin\left(\theta_{nm}^{\text{vpp}}-\delta_{ntc}^{\text{vpp}}+\delta_{mtc}^{\text{vpp}}\right)=0, \end{array}\right. \tag{30}$$

Steady-state security constraints: We consider apparent power flow

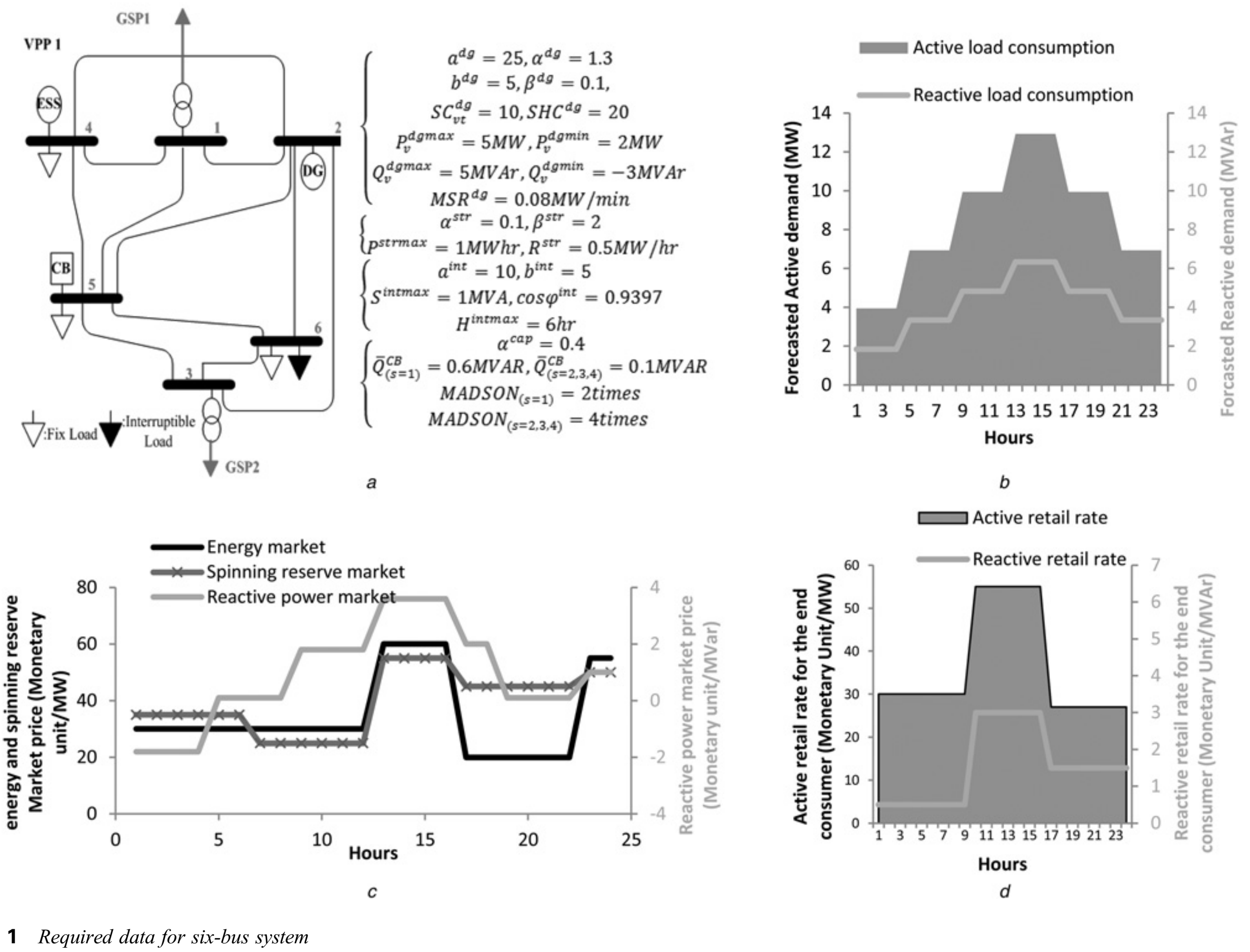


Fig. 1 *Required data for six-bus system*

a Single-line diagram
b Forecasted daily demand
c Prices of energy, spinning reserve and reactive power markets
d Retail rates for the end consumers

limit of the transmission lines and voltage limit of the buses

$$\sqrt{P_{nmc}^{\mathrm{vpp}\,2}\left(V_{ntc}^{\mathrm{vpp}}, \delta_{ntc}^{\mathrm{vpp}}\right) + Q_{nmc}^{\mathrm{vpp}\,2}\left(V_{ntc}^{\mathrm{vpp}}, \delta_{ntc}^{\mathrm{vpp}}\right)} \le F_{nm}^{\mathrm{vpp}}, \forall n, \forall m \in \phi_n^{\mathrm{vpp}}, \forall c, \forall t \qquad (31)$$

$$V_n^{\min} \le \left|V_{ntc}^{\mathrm{vpp}}\right| \le V_n^{\max}, \forall n, \forall c, \forall t \qquad (32)$$

Maximum apparent power for exchanging with the upstream network: The apparent power rating of the interconnection, the transformer capacity or the contracted capacity for exchanging power between VPP and the upstream grid through each GSP, is considered as below

$$\sqrt{\left(P_{jt}^{\mathrm{Energy}} + R_{jtc}^{\mathrm{Spinning\,Reserve}}\right)^2 + Q_{jt}^{\mathrm{Reactive}\,2}} \le F_j^{\max-\mathrm{upstream}}, \forall j \in \Psi_n^{\mathrm{vpp}}, \forall c, \forall t \qquad (33)$$

3.4 Solving the problem

The optimisation problem is a MINLP. The MINLP model is solved by the generalised algebraic modeling systems software, using the DICOPT solver. It iteratively invokes the CONOPT3 and CPLEX solvers for non-linear and mixed-integer programming solutions, respectively [34]. The resulting model is solved using a laptop computer with a 2.2 GHz processor.

4 Numerical results

In this section, we consider a six-bus system as a VPP in two cases to validate the proposed model. Then in order to evaluate the impact of markets prices on the solution of the proposed model, a sensitivity analysis is done. Moreover, we test our proposed model to a larger system (i.e. IEEE 30-bus system) to generalise the application of it.

The single-line diagram, technical parameters and cost coefficients of six-bus VPP are given in Fig. 1*a*. It is worthy of mentioning, since the spinning reserve must be provided in 10 min [18], and the ramping capability of DG is equal to $\mathrm{MSR}^{\mathrm{dg}} = 0.08$ MW/min, so the maximum capability of DG is equal to 0.8 MW (10×0.08) in spinning reserve market at each hour. Moreover, the characteristics

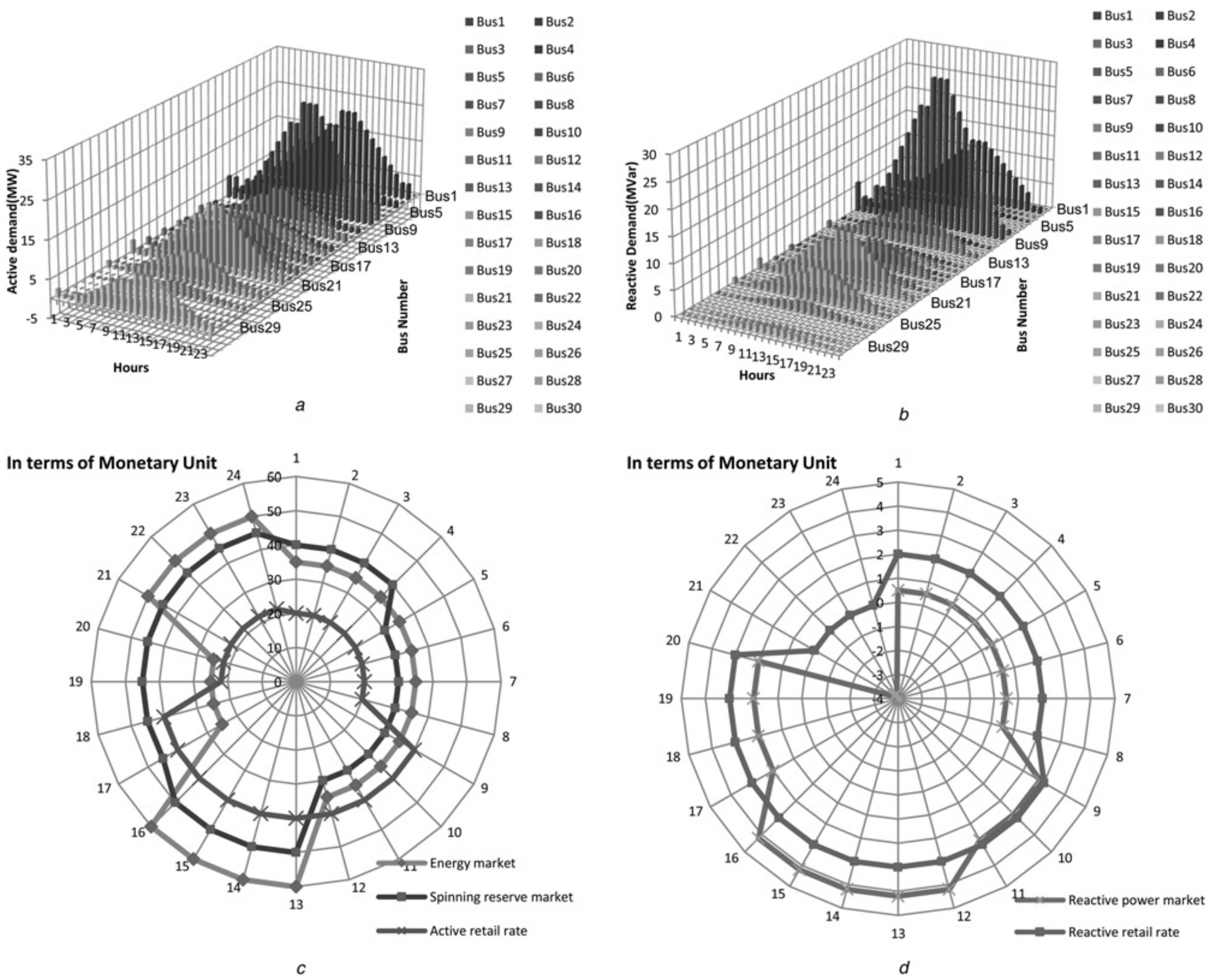


Fig. 2 *Bidding problem required data for IEEE 30-bus system*

a Forecasted daily active demand
b Forecasted daily reactive demand
c Prices of energy and spinning reserve markets, also active retail rate for the end consumers in 24 h
d Prices of reactive power market and reactive retail rate for the end consumers in 24 h

Table 2 DG units required data-1 (IEEE 30-bus system)

Bus	Unit numbers	a_{vt}^{dg}, $\frac{\text{monetary unit}}{\text{MW}}$	b_{vt}^{dg}, monetary unit	SC_{vt}^{dg}, monetary unit	SHC_{vt}^{dg}, monetary unit	α_{vt}^{dg}, $\frac{\text{monetary unit}}{\text{MVar}}$	β_{vt}^{dg}, monetary unit
1	1	31	5.5	12	20	2.6	0.21
2	2	21	5.5	12	20	1.6	0.23
22	3	23	5.5	12	20	1.4	0.25
27	4	30	5.5	12	20	1.5	0.21
23	5	41	5.5	12	20	3.75	0.22
13	6	47	5.5	12	20	3.15	0.21

Table 3 DG units required data-2 (IEEE 30-bus system)

Bus	Unit numbers	P_v^{dgmax}, MW	P_v^{dgmin}, MW	Q_v^{dgmax}, MVar	Q_v^{dgmin}, MVar	S_v^{dgmax}, MVA	MSR_v^{dg}, MW/min
1	1	40	5	15	−10	–	0.7
2	2	30	0	15	−10	31	0.5
22	3	40	0	15	−10	–	0.7
27	4	25	2	15	−10	–	0.45
23	5	15	0	15	−10	20	0.25
13	6	5	0	5	−5	–	0.1

of the transmission lines are given in [35]. The fix loads are the same, and the IL (up to 1 MVA) at bus 6 may curtail for the maximum of 6 h. Fig. 1*c* shows forecasted prices of the markets. During the hours 1–4, negative value of the reactive power price shows that ISO requires reactive power absorption. For the sake of simplicity, the penalty coefficient of power factor is considered zero at first.

A modified IEEE 30-bus system [36] has been used to show the effectiveness of the proposed model for larger VPP with multiple DGs. The VPP exchanges energy and ancillary services at bus 11 (GSP1) and bus 8 (GSP2). The required data for IEEE 30-bus system are given in Figs. 2*a*–*d*. Furthermore, technical parameters and production cost are given in Tables 2 and 3.

4.1 Case 1: without considering arbitrage opportunities

In this case, we assume that the six-bus VPP participates in energy and reactive power markets, so it can only purchase in energy and reactive power markets to supply the end consumers. Therefore, VPP operates none of DG, ESS, IL and CB. The exchanged power with the upstream network is shown in Figs. 3*a* and *c*. In addition, Figs. 3*a* and *c* illustrates the bidding of VPP to the energy and reactive power market from the GSPs. VPP bids from GSP2 more than GSP1 to minimise its power losses. To investigate this issue, in Figs. 3*b* and *d*, active and reactive power losses of VPP in three cases are shown. As shown, the power losses of VPP in case of exchanging power through both GSP1 and GSP2 are the lowest.

Table 4 shows payment to the markets, revenues and profits of VPP in this case. The expected profit of VPP reaches to 405.69 monetary unit.

4.2 Case 2: considering arbitrage opportunities

In this case, VPP participates in the energy, spinning reserve and reactive power markets simultaneously. For the sake of simplicity, we assume that VPP can only provide spinning reserve service in GSP2. Moreover, VPP operates DG, ESS, IL and CB to gain more profit.

In Fig. 4, the state of VPP components are shown. Fig. 4*a* represents that during the hours 3–4 and 21–22 in which the energy market prices are low, the storage is fully charged. Then it is discharged during the hours 13–14 and 23–24 in which the market price is high. Owing to the cost of ESS operation (16.4 monetary unit), the arbitrage strategy of ESS leads to an increase of the profit of VPP by 48.6 monetary unit.

Fig. 4*b* illustrates the IL curtailment options. During the hours 19–22, the price of spinning reserve market is higher than the energy market price. Therefore, considering the cost of curtailing (14.40 monetary unit/h) and the retail energy rate of VPP, more profit can be obtained by curtailing the IL in the spinning reserve market, so the arbitrage opportunity between energy and spinning reserve services is explored. Moreover, in the last 2 h, the price of energy market is higher than the spinning reserve price. Considering the cost of curtailing and the retail energy rate of VPP, the maximum permitted value of load is curtailed to sell in the energy market during these hours.

DG active power generations associated to the energy and the spinning reserve markets are shown in Fig. 4*c*. In the first 16 h and the last 2 h, the cost of DG to generate active power is lower than the prices of the energy market, so the DG is switched on. In the first 6 h, the price of the energy market is lower than that of the spinning reserve market, so an arbitrage opportunity exists between energy and spinning reserve services. Therefore, the maximum capability of DG is traded in the spinning reserve market, and the rest of DG capacity (4.2 MW) is allocated to the energy market. Since the price of the energy market is greater than that of the spinning reserve market during the hours 7–16 and 23–24, the whole capacity of DG is traded in the energy market. During the hours 17–22, the price of the energy market is very low, but the price of the spinning reserve market is so high that it is profitable for DG to be on and operate at its minimum output in order to use its capability for the spinning reserve service provision.

Reactive power generations and apparent powers of DG are shown in Fig. 4*d*. In the first 4 h, the cost of DG to produce reactive power is lower than the price of reactive power market. Therefore, the maximum absorption of reactive power is traded in the reactive power market. During the hours 5–8 and 19–24, reactive power market price is lower than the reactive power injection cost of DG, so reactive power is not generated. During the hours 9–18, the production cost of DG is lower than the price of reactive power market, so maximum reactive power injection should be traded in the reactive power market. However, the prices of reactive power market are much lower than that of the energy and spinning reserve markets, so the active power generation is much more profitable than the reactive power generation for VPP. Hence, first, maximum capacity of DG is traded in the energy and spinning reserve markets, and then the maximum apparent power limits the reactive power injection of DG. As a result, the maximum apparent power limits the injection of reactive power to +4.1 MVar during the hours 9–16. Moreover, during the hours 17–18, the apparent power limit of DG is not reached, so DG generates the maximum reactive power (+5 MVar).

During the hours 9–18 and 23–24, the cost of CB operation is lower than the price of reactive power market. However, given that the first step of CB can only be switched two times, the first step stays switched on during the hours19–22 till all steps of CB are able to provide reactive power service during the hours 23–24; see Fig. 4*e*. Therefore, an increase of the expected profit of VPP is reached.

Fig. 5*a* shows the aggregated profiles including active power generation, consumption, total active power exchanged with the main grid and bidding from each GSP. The active power generation associated with the energy market [curve (1)] includes DG generation, load curtailment and storage discharging for the energy market. Moreover, the total active power generation associated with the spinning reserve market [curve (2)] is the summation of DG generation and load curtailment for the spinning reserve market. Curve (3) shows the total exchanged power with the main grid from the GSPs associated with the energy market, and the positive value indicates that the power has been absorbed by VPP. For satisfying the bilateral contracts, during the hours 1–2, VPP supplies whole of the end consumers by local generation [curve (1)] and injects excess generation into the energy market; however, during the hours 3–16 and 23–24, VPP supplies part of the end consumers by local generation [curve (1)] and absorbs lack of generation from the energy market. Noteworthily that during the hours 17–22, VPP has to generate active power in the energy market to be able to provide spinning reserve by DG, and therefore it supplies remnant of the end consumers by purchasing from energy market during these hours. As a result, VPP satisfies the bilateral contracts either by local generation or by purchases

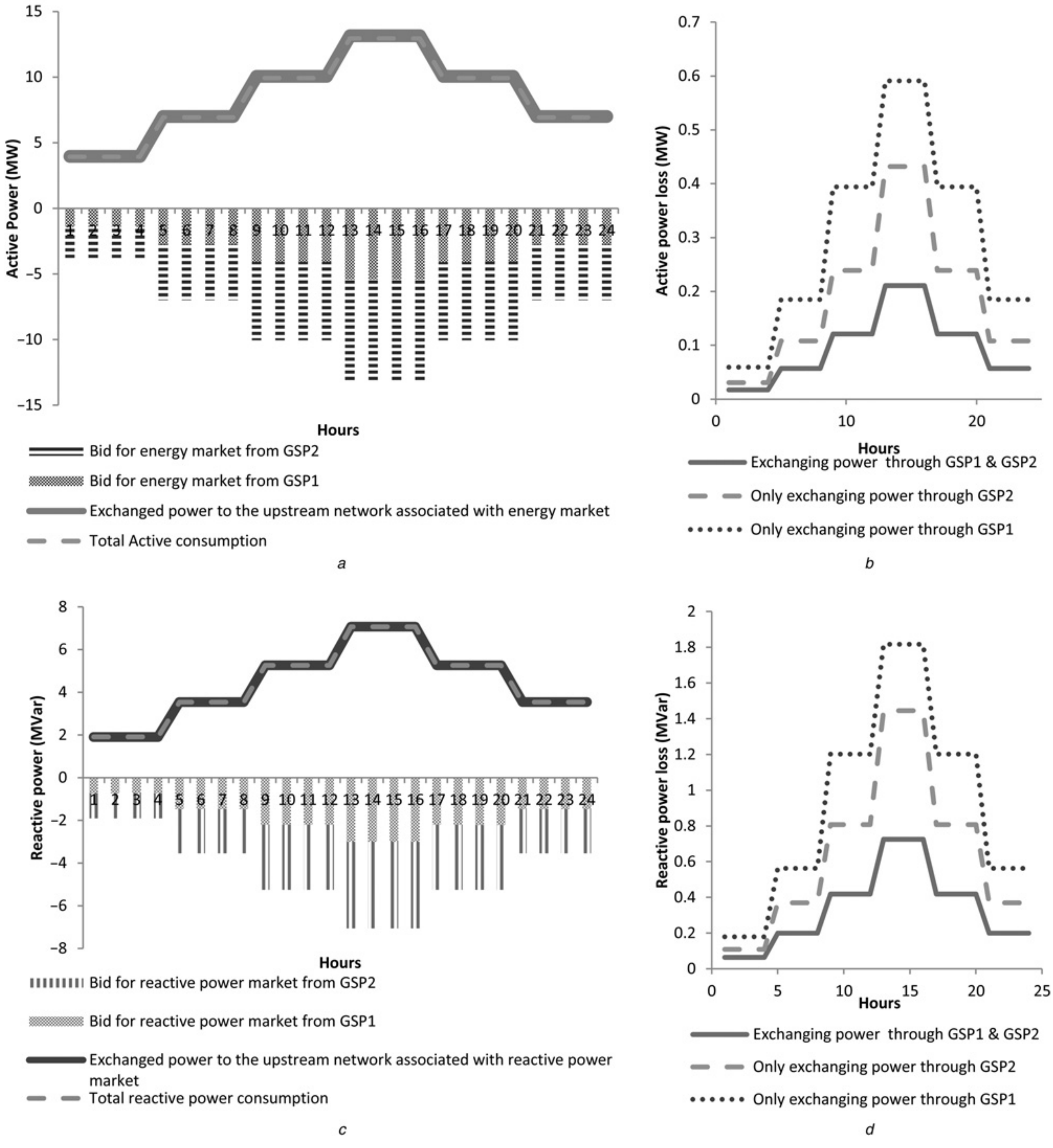


Fig. 3 *Results of VPP bidding problem without considering arbitrage opportunities*

a Total active power consumption, exchanging active power and bidding to the energy market from two GSPs
b Comparison of active power losses in cases of exchanging power through one or two GSPs
c Total reactive power consumption, exchanging reactive power and bidding to the reactive power market from two GSPs
d Comparison of reactive power losses in cases of exchanging power through one or two GSPs

Table 4 Payment, revenue and profit of VPP – case 1

Hours	Energy market payment, monetary unit		Reactive power market payment, monetary unit		Revenue of supplying active power of the end consumer, monetary unit	Revenue of supplying reactive power of the end consumer, monetary unit	Net profit, monetary unit
	GSP1	GSP2	GSP1	GSP2			
1	−42.22	−76.50	1.30	2.13	118.19	0.92	3.83
2	−42.22	−76.50	1.30	2.13	118.19	0.92	3.83
3	−42.22	−76.50	1.30	2.13	118.19	0.92	3.83
4	−42.22	−76.50	1.30	2.13	118.19	0.92	3.83
5	−82.38	−127.52	−0.15	−0.21	208.19	1.67	−0.39
6	−82.38	−127.52	−0.15	−0.21	208.19	1.67	−0.39
7	−82.38	−127.52	−0.15	−0.21	208.19	1.67	−0.39
8	−82.38	−127.52	−0.15	−0.21	208.19	1.67	−0.39
9	−122.51	−179.31	−3.98	−5.49	298.19	2.42	−10.67
10	−122.51	−179.31	−3.98	−5.49	546.68	14.53	249.93
11	−122.51	−179.31	−3.98	−5.49	546.68	14.53	249.93
12	−122.51	−179.31	−3.98	−5.49	546.68	14.53	249.93
13	−325.06	−463.96	−10.80	−14.64	711.68	19.03	−83.76
14	−325.06	−463.96	−10.80	−14.64	711.68	19.03	−83.76
15	−325.06	−463.96	−10.80	−14.64	711.68	19.03	−83.76
16	−325.06	−463.96	−10.80	−14.64	711.68	19.03	−83.76
17	−81.89	−119.32	−4.42	−6.10	268.37	7.26	63.90
18	−81.89	−119.32	−4.42	−6.10	268.37	7.26	63.90
19	−81.29	−119.92	−0.22	−0.30	268.37	7.26	73.90
20	−81.29	−119.92	−0.22	−0.30	268.37	7.26	73.90
21	−54.93	−85.00	−0.15	−0.21	187.37	5.01	52.10
22	−54.93	−85.00	−0.15	−0.21	187.37	5.01	52.10
23	−151.28	−233.53	−1.47	−2.08	187.37	5.01	−195.97
24	−151.28	−233.53	−1.47	−2.08	187.37	5.01	−195.97
total	−3027.43	−4504.70	−67.04	−90.19	7913.47	181.58	405.69

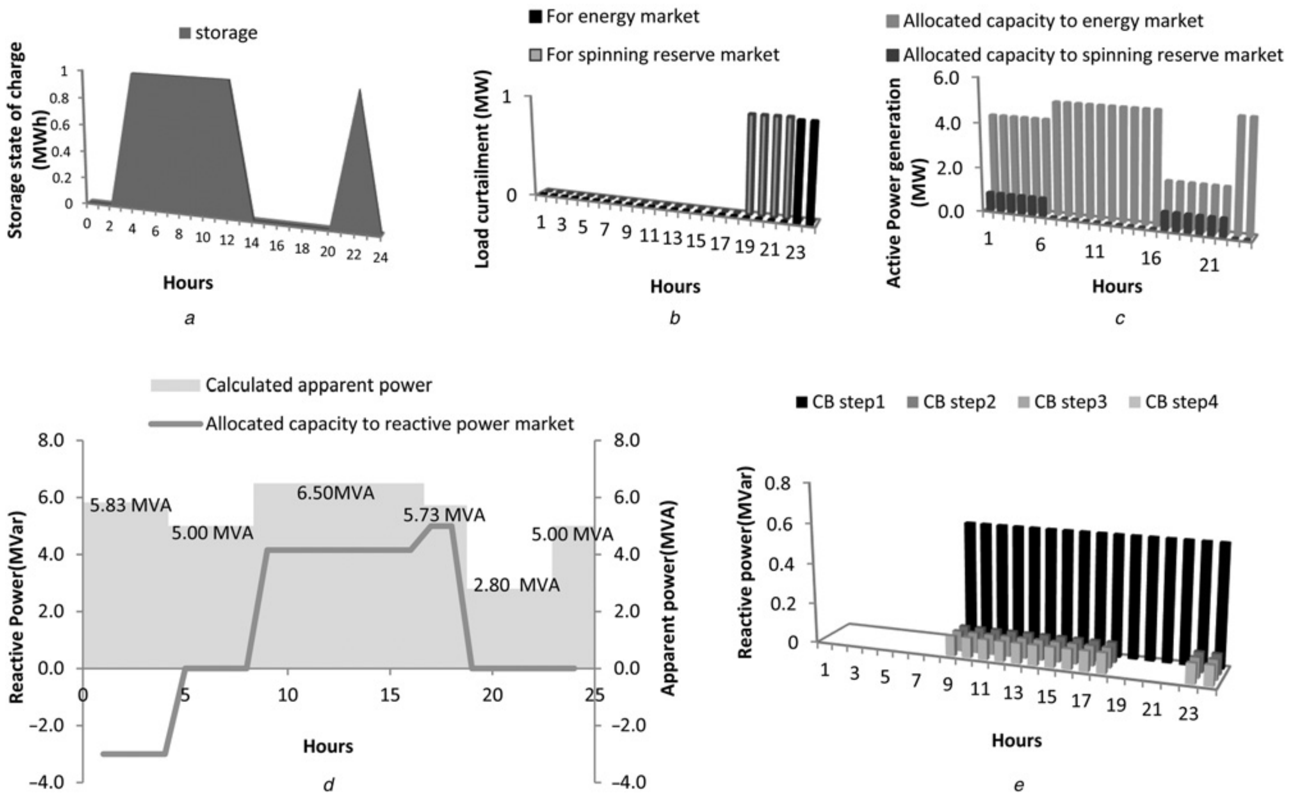
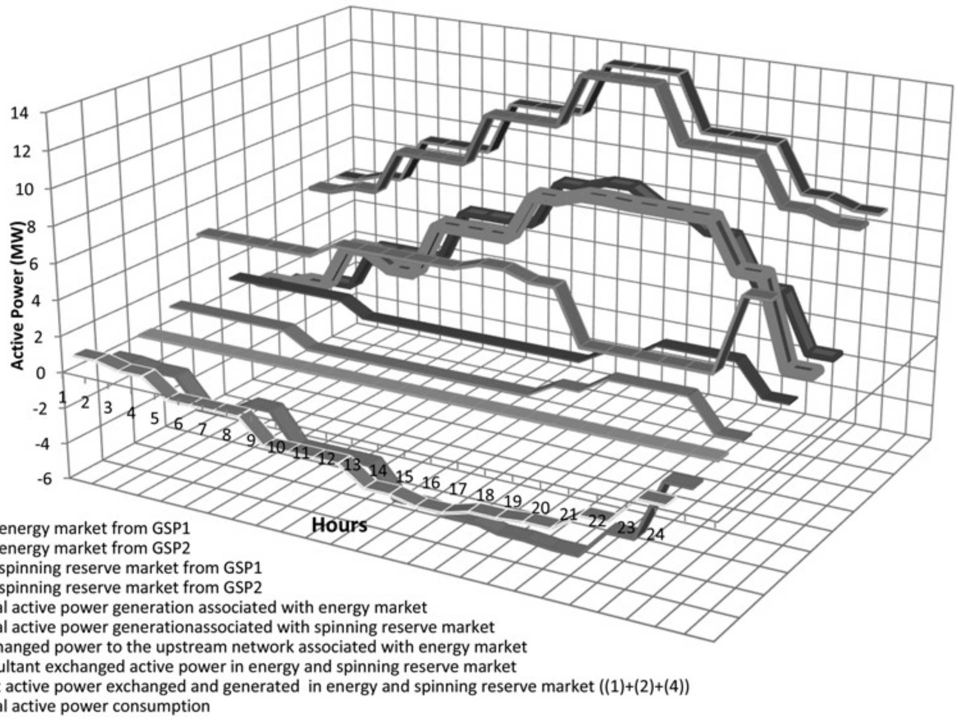


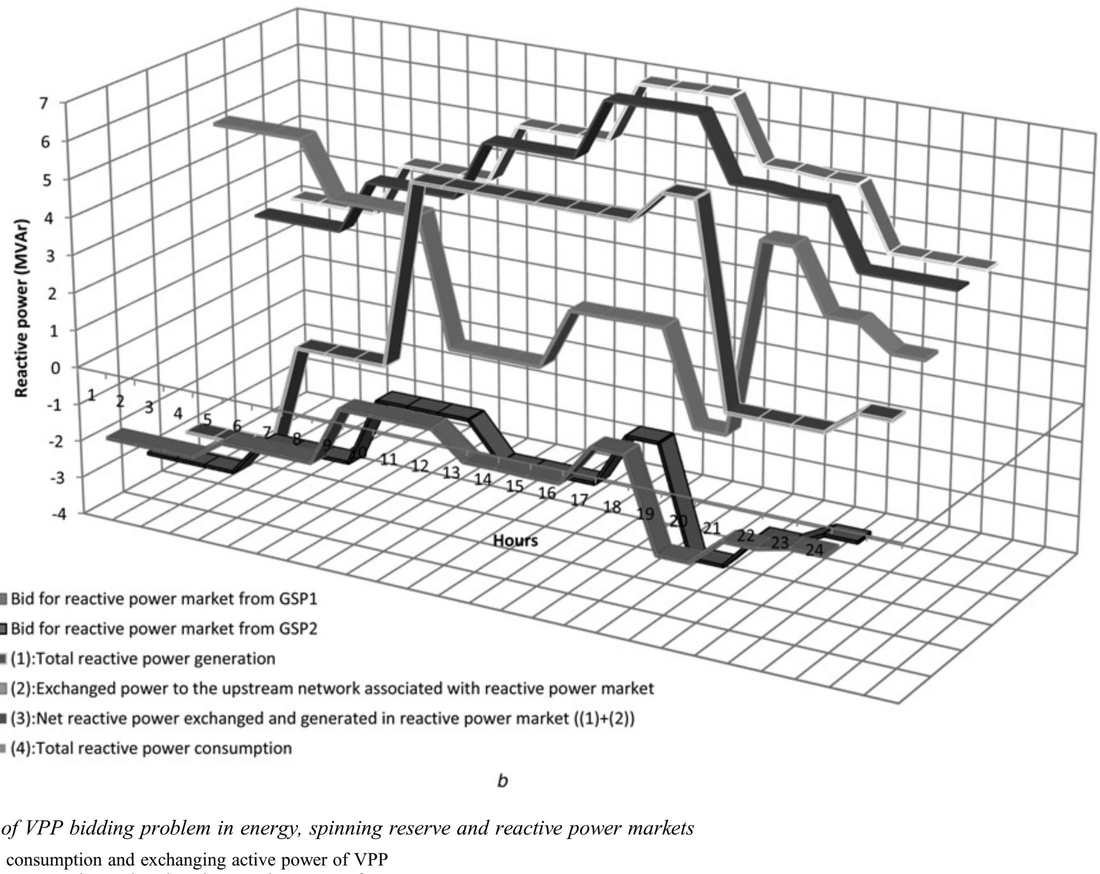
Fig. 4 *VPP components operation state*

a Charging and discharging of storage
b Load interrupting options
c Allocated capacity of DG unit for energy and spinning reserve markets
d Allocated capacity of DG unit for reactive power market and calculated apparent power
e Switched-on CB steps

from the energy market to increase its profit by arbitrage of bilateral contracts. Curve (4) shows the result of total exchanged power with the main grid from the GSPs considering the energy and spinning reserve markets. The difference between curve (3) and curve (4) is equal to the total value of spinning reserve provided by VPP. During the hours 1–6 and 17–22, in which the price of the



a



b

Fig. 5 *Results of VPP bidding problem in energy, spinning reserve and reactive power markets*

a Total generation, consumption and exchanging active power of VPP
b Total generation, consumption and exchanging reactive power of VPP

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Table 5 Revenue, cost and profit of VPP – case 2

Hours	Energy market revenue, monetary unit		Spinning reserve market revenue, monetary unit		Reactive power market revenue, monetary unit		Revenue of supplying active power of the end consumer considering IL, monetary unit	Revenue of supplying reactive power of the end consumer considering IL, monetary unit	Active and reactive powers production cost of DG + load curtailment cost + ESS operation cost + operation cost of CB, monetary unit	Net profit, monetary unit
	GSP1	GSP2	GSP1	GSP2	GSP1	GSP2				
1	15.92	−9.95	0	28	3.63	5.43	118.19	0.92	144	18.14
2	15.92	−9.95	0	28	3.63	5.43	118.19	0.92	134	28.14
3	6.76	−14.29	0	28	3.63	5.42	118.19	0.92	138.05	10.58
4	6.76	−14.29	0	28	3.63	5.42	118.19	0.92	136.05	12.58
5	−19.79	−64.14	0	28	−0.15	−0.21	208.19	1.67	135	18.57
6	−19.79	−64.14	0	28	−0.15	−0.21	208.19	1.67	130	23.57
7	−19.79	−40.14	0	0	−0.15	−0.21	208.19	1.67	130	19.57
8	−19.79	−40.14	0	0	−0.15	−0.21	208.19	1.67	130	19.57
9	−57.18	−94.11	0	0	0.22	−0.39	298.19	2.42	135.86	13.30
10	−57.18	−94.11	0	0	0.22	−0.39	546.68	14.53	135.86	273.89
11	−57.18	−94.11	0	0	0.22	−0.39	546.68	14.53	135.86	273.89
12	−57.18	−94.11	0	0	0.22	−0.39	546.68	14.53	135.86	273.89
13	−174.87	−283.60	0	0	−2.04	−4.28	711.68	19.03	139.91	126.00
14	−174.87	−283.60	0	0	−2.04	−4.28	711.68	19.03	137.91	128.00
15	−193.25	−292.73	0	0	−2.12	−4.29	711.68	19.03	140.86	97.46
16	−193.25	−292.73	0	0	−2.12	−4.29	711.68	19.03	135.86	102.46
17	−57.06	−103.93	0	36	1.01	0.43	268.37	7.26	81.96	70.12
18	−57.06	−103.93	0	36	1.01	0.43	268.37	7.26	81.96	70.12
19	−54.94	−105.50	0	78.29	−0.19	−0.26	243.00	7.26	91.64	76.02
20	−54.94	−105.50	0	78.29	−0.19	−0.26	243.00	7.26	89.64	78.02
21	−34.76	−73.85	0	78.29	−0.12	−0.17	162.00	5.01	93.69	42.72
22	−34.76	−73.85	0	78.29	−0.12	−0.17	162.00	5.01	91.69	44.72
23	−11.00	−21.69	0	0	−0.97	−1.26	162.00	4.50	153.81	−22.23
24	−11.00	−21.69	0	0	−0.97	−1.26	162.00	4.50	146.81	−15.23
total	−1314.30	−2396.08	0	553.16	5.95	−0.35	7761.24	180.55	3006.28	1783.90

spinning reserve market is greater than that of energy market, the arbitrage opportunities are realised between energy and spinning services. The summations of curve (1), curve (2) and curve (3) indicate the supply curve of VPP [curve (4)]. Moreover, curve (5) shows the VPP's total active power consumption, composed of the forecasted active load, the active power losses and the charged capacity of storage. During all hours, curve (4) is exactly the same as curve (5), which indicates the active power supply–demand balancing constraint of VPP. Finally, Fig. 5*a* shows the bidding of VPP to the energy and the spinning reserve markets for GSP1 and GSP2. In this figure, the negative values show the bids for purchasing power, and positive values correspond to the bids for selling power to the markets.

The aggregated profiles including reactive power generation, reactive power consumption, total exchanging of the reactive power with the main grid and bidding to the reactive power market from the GSPs are illustrated in Fig. 5*b*. The total reactive power generation curve includes DG generation and the amount of switched-on CB. Curve (2) shows the total reactive power exchanged with the main grid from the GSPs, and the positive value indicates that the power is absorbed by VPP. As a result, the arbitrage between local generation and purchases from the market would be more profitable than purchases from the market alone. Curve (4) shows the VPP's total reactive power consumption including the forecasted reactive load and the reactive power loss. During all hours, the matching of curve (3) and curve (4) indicates the reactive power supply–demand balancing constraint of VPP. Finally, Fig. 5*b* shows the bidding of VPP to the reactive power market from GSP1 and GSP2. In this figure, the negative values show the bids for purchasing (i.e. when VPP absorbs reactive power) or selling (i.e. when ISO requires the absorption of reactive power) the absorption of reactive power, and the positive values correspond to the bids for selling the injection of reactive power (i.e. when VPP injects reactive power) into the market.

Table 5 shows the costs, revenues and profits of VPP in this case. The maximum expected profit of VPP reaches to 1783.90 monetary unit. By described arbitrage opportunities, VPP increases its profit to 1783.90–405.69 = 1378.21 monetary unit.

Table 6 Impact of power factor penalty

Penalty coefficient, $\alpha_{jt}^{\mathrm{PF}}\left(\frac{\text{monetary unit}}{\text{MVar}}\right)$		0	2	4
total reactive power exchanged with upstream network, MVar h	GSP1	−22.26	−10.10	−6.28
	GSP2	−37.13	−22.90	−18.41
	total	−59.39	−33.00	−24.69
total penalty, monetary unit		0	7.954	0
net profit, monetary unit		1783.90	1761.93	1755.04

4.3 Sensitivity analysis

4.3.1 Impact of the power factor penalty: To evaluate the impact of power factor penalty, the penalty coefficient has been increased from 0 to 4 monetary unit/MVar step by step. By increasing the penalty coefficient, the VPP profit and the total reactive power exchanged with the upstream network have been decreased; see Table 6. When the penalty coefficient is greater than the price of reactive power market, the VPP tries to eliminate its power factor penalty in each hour by decreasing the reactive power exchanged with the upstream network. Therefore, the total penalty decreases to zero in the last case which the maximum price of reactive power market (3.6 monetary unit/MVar is less than the penalty coefficient ($\alpha_{jt}^{\mathrm{PF}} = 4$).

4.3.2 Impact of the forecasted prices: The six-bus system is applied considering two scenarios for spinning reserve market prices, where each scenario includes three levels for the prices of energy, spinning reserve and reactive power markets; see Fig. 6. As a result, the expected profit is calculated for each one of 27 states. The probabilities of low, middle and high level are equal to 0.3, 0.5 and 0.2, respectively.
Considering the first scenario: In this scenario, the inversion between prices of energy and spinning reserves is not considered (i.e. there is no arbitrage opportunity between energy and spinning reserves); see Fig. 6. The price curves of energy and reactive power markets shown in Fig. 1*c* are considered as the middle prices in this case. In Table 7, the profit of VPP is shown in each state. VPP provides no spinning reserve service, so the profit stays constant by changing the price of spinning reserve market. As it shown before in Section 4.2, VPP is often a consumer in the

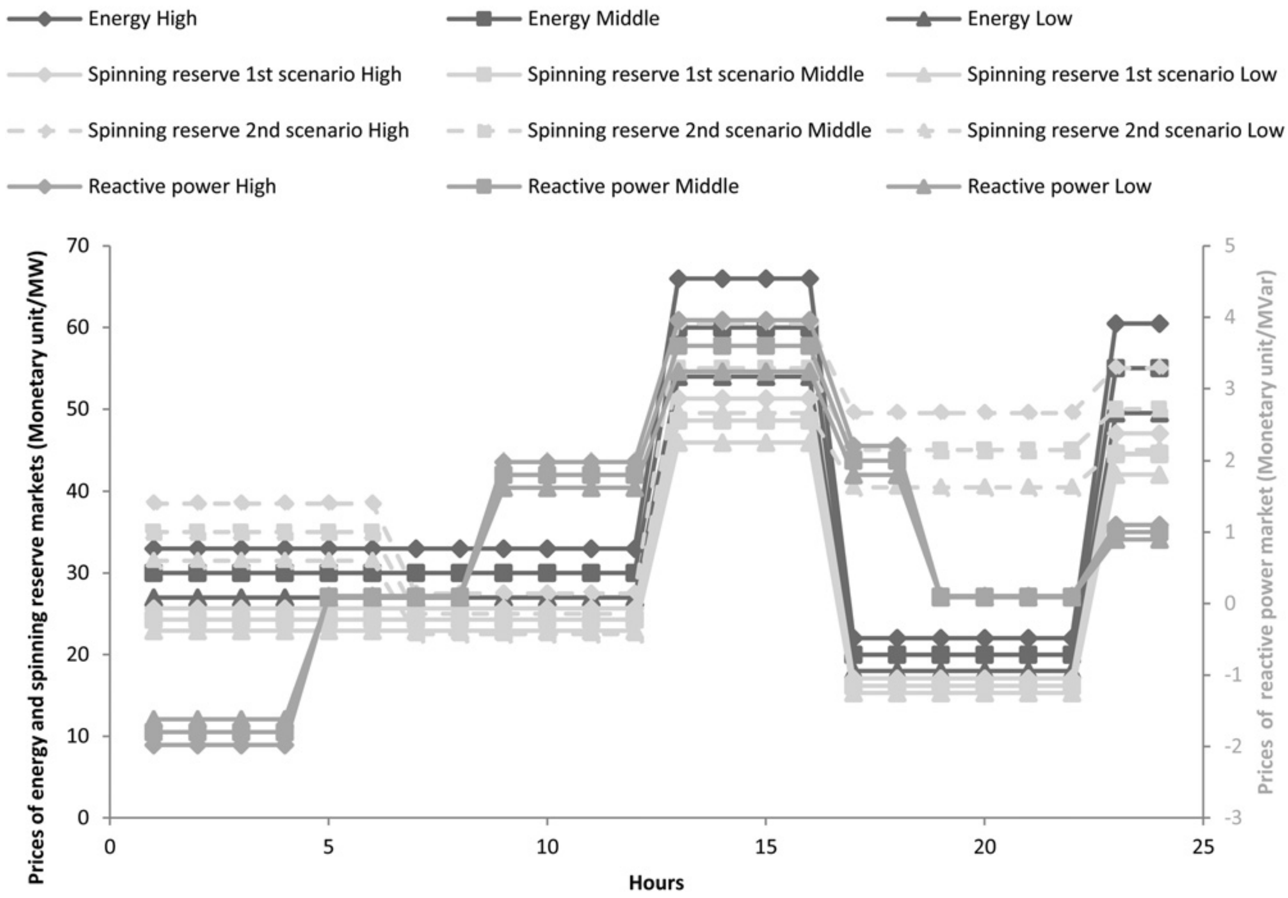

Fig. 6 *Three levels of prices: curves of energy, spinning reserve and reactive power markets*

energy market, so the profit of VPP has an inverse relationship with the energy market prices.

In this case, providing no spinning reserve service caused the DG is turned off during the hours 17–22 (it was described in Section 4.2), so VPP is often a consumer (purchaser) in the reactive power market. Therefore, the VPP profit decreases by increasing the prices of reactive power market.

Finally, according to Table 7, the expected profit of VPP is equal to 1747.19.

Considering the second scenario: In this scenario, the price curves of energy, spinning reserve and reactive power markets shown in Fig. 1*c* are considered as middle prices level. VPP provides spinning reserve, when the prices of spinning reserve market are greater than that of energy market during the hours 1–6 and 17–22 except in low prices level of energy and spinning reserve markets (first row in Table 8), in which the prices are not high enough to be profitable for VPP to provide spinning reserve service. Hence, the VPP profit increases by increasing the spinning reserve market price. As mentioned before, VPP is often a consumer in the energy market, so the profit of VPP has an inverse relationship with deviation of energy market prices.

In this scenario, in low prices level of energy and spinning reserve markets (first row in Table 8), providing no spinning reserve service caused the DG is turned off during the hours 17–22, so the VPP profit decreases by increasing the prices level of reactive power market as well as Section 4.3.1. However, in the other prices level of energy and spinning reserve markets, VPP is often a producer (seller) in the reactive power market. Thus, the VPP profit increases by increasing the prices level of reactive power market.

Finally, according to Table 8, the expected profit of VPP is equal to 1823.63. In comparison of these two scenarios, the expected profit of VPP increases by using arbitrage opportunities between energy and spinning reserve services.

Table 7 Profit of VPP – first scenario

The price level of energy market	The price level of spinning reserve market	The price level of reactive power market		
		Low	Middle	High
low	low	2086.95	2085.48	2084.02
	middle	2086.95	2085.48	2084.02
	high	2086.95	2085.48	2084.02
middle	low	1706.10	1704.63	1703.16
	middle	1706.10	1704.63	1703.16
	high	1706.10	1704.63	1703.16
high	low	1346.42	1345.47	1344.59
	middle	1346.42	1345.47	1344.59
	high	1346.42	1345.47	1344.59
		expected profit		1747.19

Table 8 Profit of VPP – second scenario

The price level of energy market	The price level of spinning reserve market	The price level of reactive power market		
		Low	Middle	High
low	low	2108.55	2107.08	2106.35
	middle	2159.26	2159.75	2160.24
	high	2251.97	2252.46	2252.95
middle	low	1734.12	1734.67	1735.23
	middle	1783.38	1783.90	1784.49
	high	1840.29	1840.85	1841.41
high	low	1391.39	1392.46	1393.60
	middle	1422.59	1423.66	1424.80
	high	1467.72	1468.21	1468.77
		expected profit		1823.63

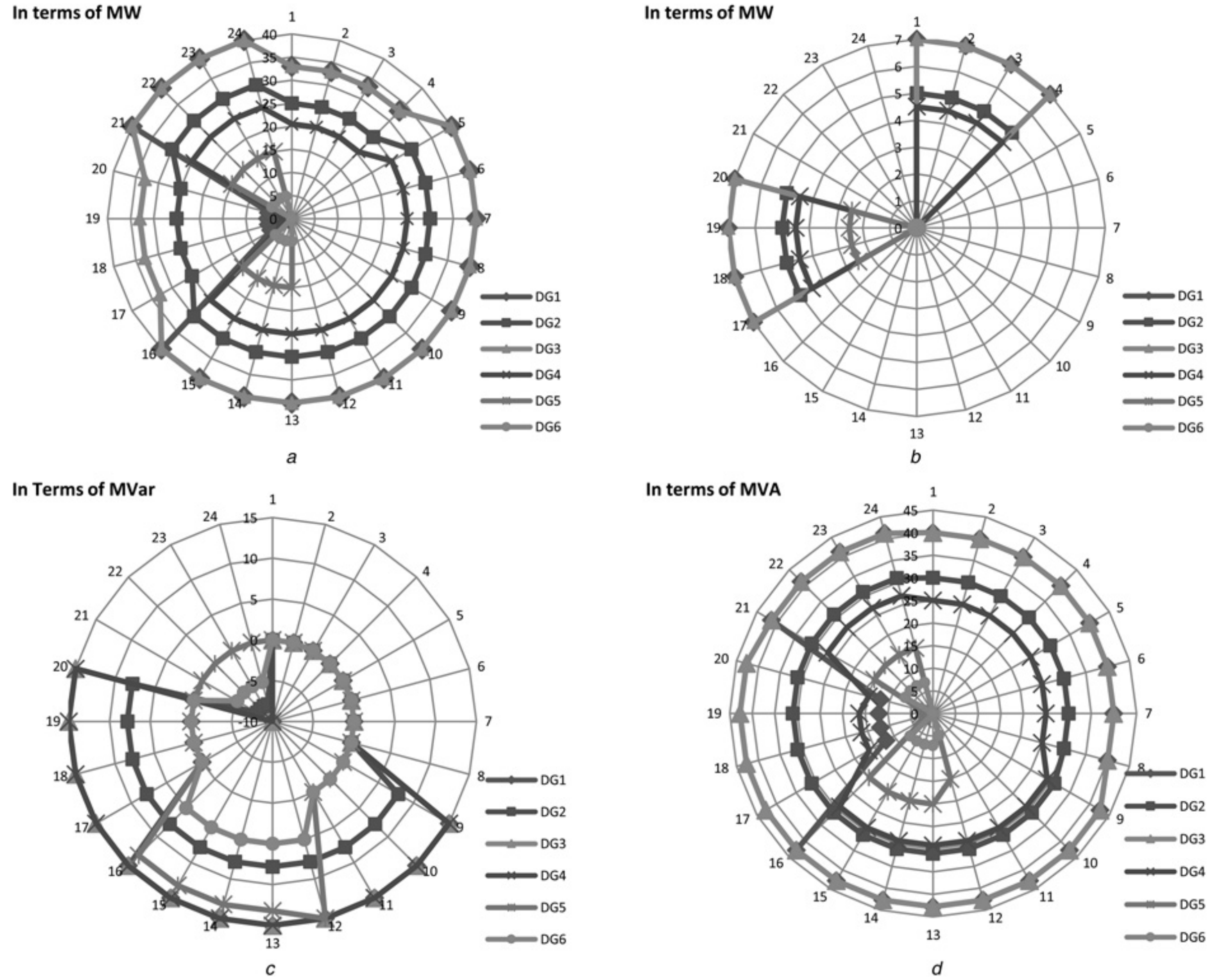

Fig. 7 *Results of proposed model for IEEE 30-bus system*

a Allocated capacity of DG units for energy market in 24 h
b Allocated capacity of DG units for spinning reserve market in 24 h
c Allocated capacity of DG units for reactive power market in 24 h
d Calculated apparent power of DG units in 24 h

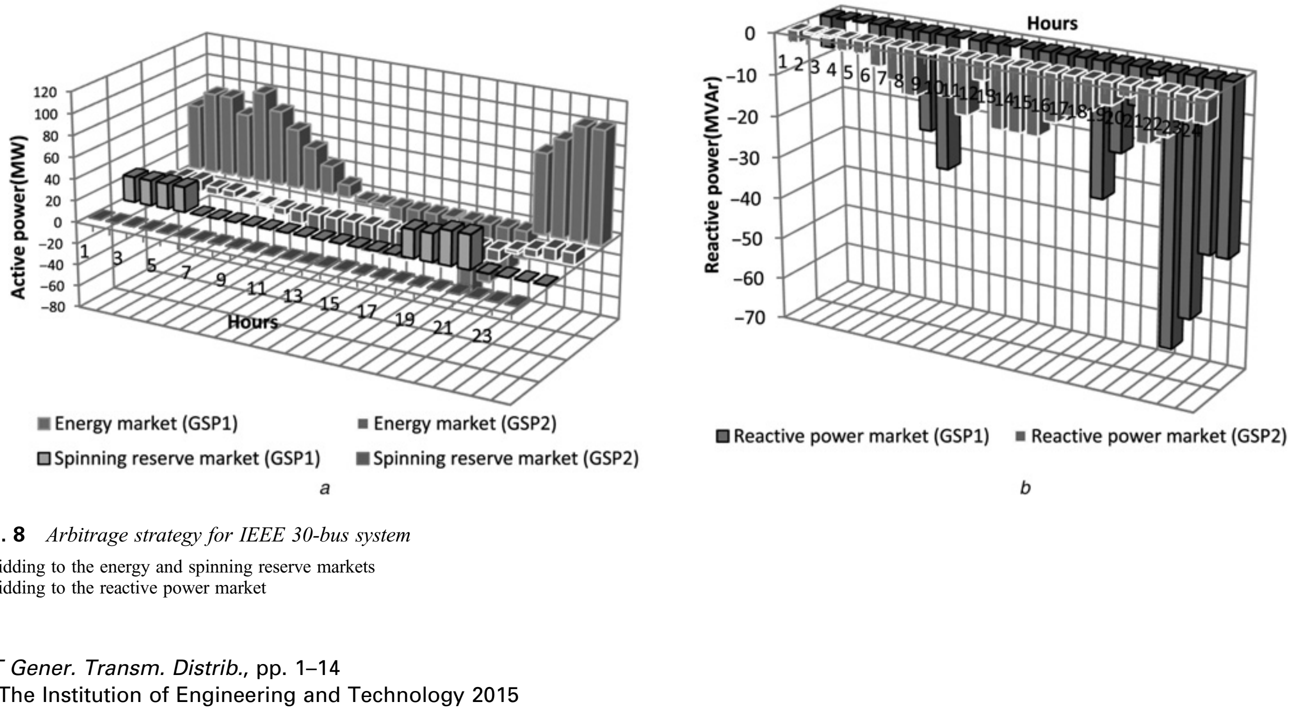

Fig. 8 *Arbitrage strategy for IEEE 30-bus system*

a Bidding to the energy and spinning reserve markets
b Bidding to the reactive power market

4.4 IEEE 30-bus system

The results of solving proposed model in case of IEEE 30-bus system are shown in Figs. 7 and 8. Moreover, the allocated active power capacities of DG units to energy and spinning reserve markets are shown in Figs. 7*a* and *b*, respectively. Generated reactive and apparent powers of DGs are shown in Figs. 7*c* and *d*. Finally, Figs. 8*a* and *b* show bidding of VPP to the energy, spinning reserve and reactive power markets from GSP1 and GSP2. The maximum expected profit of VPP reaches to 19,655 Monetary Unit. Though more complexities due to simultaneous management of active and reactive powers are added to the model, the computational time is about 234 s.

5 Conclusions

This paper represents the arbitrage strategy of VPP by optimal bidding in energy, spinning reserve and reactive power markets from multiple GSPs. A six-bus system was used to evaluate the presented model, and two sensitivity analyses are done to investigate the impact of the markets prices and power factor penalty on the solution of the proposed model. Moreover, a modified IEEE 30-bus system has been used to show the effectiveness of the proposed model for larger VPP with multiple DERs. At first, participation of VPP without considering arbitrage opportunities in energy and reactive markets was investigated. The results showed that VPP is an entity with role of consumer, so that it can provide energy and reactive power services from multiple GSPs by minimising its active power losses. Then participation in energy, spinning reserve and reactive power markets with considering arbitrage opportunities was investigated. The results show that VPP can provide energy, spinning reserve service and reactive power service, regarding the economical and technical aspects. VPP has two roles including consumer and producer in energy and reactive power markets. Furthermore, VPP can participate in spinning reserve market by providing this service.

VPP discovers three kinds of arbitrage opportunities by the proposed SCPBUC to increase its profit. First, VPP optimises energy and spinning reserve services simultaneously, so it can create the arbitrage between energy and spinning reserve services. Second, VPP satisfies bilateral contracts with end consumers either by local generation or by purchases from the energy and reactive power markets. Third, VPP can purchase (charge) energy when electricity prices are low and sell (discharge) energy when electricity prices are high by ESSs. Significantly, the expected profit can increase by arbitrage strategy of VPP.

6 References

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