Example 7.3 explanation

The parameters of a DFIG are given in Table 1. The stator to rotor turns ratio is unity.

Parameter	Value	Parameter	Value
Rated frequency	60 Hz	Stator resistance (R_s)	2 mΩ
Rated voltage (line-line rms)	690 V	Rotor resistance (R_r)	1.5 mΩ
Number of poles (p)	6	Stator leakage reactance (X_{ls})	50 mΩ
Full load slip	1%	Rotor leakage reactance (X_{lr})	47 mΩ
		Magnetizing reactance (X_m)	860 mΩ

The stator voltage oriented frame is chosen for the example. The angle θ_{da} in stator voltage vector frame is equal to the angle of the stator voltage vector. It can be calculated as follows.

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{0.1}$$

$$\theta_{da} = \tan^{-1}\frac{v_\beta}{v_\alpha} \tag{0.2}$$

Once the angle is computed, the stator and rotor voltages can be converted from the abc to the dq frame using following equation.

$$\begin{bmatrix} X_d \\ X_q \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta_{da} & \cos(\theta_{da} - \frac{2\pi}{3}) & \cos(\theta_{da} + \frac{2\pi}{3}) \\ -\sin\theta_{da} & -\sin(\theta_{da} - \frac{2\pi}{3}) & -\sin(\theta_{da} + \frac{2\pi}{3}) \end{bmatrix} \times \begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} \tag{0.3}$$

The flux and the current vectors are related as follows.

$$\boldsymbol{\lambda} = \mathbf{LI} \tag{0.4}$$

The vectors $\boldsymbol{\lambda}$ and $\mathbf{I}$ are given as $\boldsymbol{\lambda} = \begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \\ \lambda_{rd} \\ \lambda_{rq} \end{bmatrix}$ and $\mathbf{I} = \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix}$. The subscripts s and r are for the stator and the rotor respectively while subscripts d and q are for the d-axis and q-axis respectively.

The inductance matrix $\mathbf{L}$ in the dq frame is given as $\mathbf{L} = L_{\text{m,1-phase}} \begin{bmatrix} 1+k_{ls} & 0 & 1 & 0 \\ 0 & 1+k_{ls} & 0 & 1 \\ 1 & 0 & 1+k_{lr} & 0 \\ 0 & 1 & 0 & 1+k_{lr} \end{bmatrix}$

where

$L_{\text{m,1-phase}}$ = per phase magnetizing inductance of the induction machine,

k_{ls} = ratio of L_{ls} (stator leakage inductance) and $L_{\text{m,1-phase}}$ and
k_{lr} = ratio of L_{lr} (rotor leakage inductance) and $L_{\text{m,1-phase}}$.
The current ($\mathbf{I}$), voltage ($\mathbf{V}$) and flux ($\boldsymbol{\lambda}$) vectors are related by following equations.

$$\mathbf{V} = \mathbf{R}\mathbf{I} + \frac{d\boldsymbol{\lambda}}{dt} + \mathbf{M}\boldsymbol{\lambda} \tag{0.5}$$

where the vector $\mathbf{V}$ is given as $\mathbf{V} = \begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{rd} \\ v_{rq} \end{bmatrix}$. The resistance matrix $\mathbf{R}$ is given as

$\mathbf{R} = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix}$, where R_s and R_r are stator and rotor per phase resistances respectively.

The rotation matrix $\mathbf{M}$ occurs due the cross coupling between the d and q axes quantities. It is given as

$\mathbf{M} = \begin{bmatrix} 0 & -\omega_d & 0 & 0 \\ \omega_d & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega_{dA} \\ 0 & 0 & \omega_{dA} & 0 \end{bmatrix}$.

The electromagnetic torque τ_{em} is related to the flux, currents, load torque (τ_L) and mechanical speed is given as

$$\tau_{em} = \frac{p}{2} L_{\text{m,1-phase}} (i_{sq} i_{rd} - i_{sd} i_{rq}) = J \frac{d\omega_{mech}}{dt} + \tau_L = J \frac{2}{p} \frac{d\omega_m}{dt} + \tau_L \tag{0.6}$$

It should be noted that the angular speed ω_d mentioned in matrix $\mathbf{M}$ equals the stator voltage frequency, while ω_{dA} is given as

$$\omega_{dA} = \omega_d - \omega_m \tag{0.7}$$

The dq quantities can be converted to abc frame by following equation.

$$\begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta_{da} & -\sin\theta_{da} \\ \cos(\theta_{da} - \frac{2\pi}{3}) & -\sin(\theta_{da} - \frac{2\pi}{3}) \\ \cos(\theta_{da} + \frac{2\pi}{3}) & -\sin(\theta_{da} + \frac{2\pi}{3}) \end{bmatrix} \times \begin{bmatrix} X_d \\ X_q \end{bmatrix} \tag{0.8}$$

Equations (1.1) to (1.8) give the mathematical model of a DFIG with stator and rotor voltages as inputs and currents, electromagnetic torque and speed as outputs. These equations have been used in to build the subsystems named ‘DFIG’ and ‘Estimator’ in the Simulink model for the example. It should be noted, that in the example, the speed control loop has not been implemented. Instead, it is assumed that the speed of the machine is constant since it is coupled mechanically to the Wind turbine which has a very large inertia and it doesn’t change its speed in a time period of a few seconds.

The q-axis rotor current is controlled to regulate the stator flux, while the d-axis rotor current is controlled to regulate the active power. The active power to d-axis rotor current relation is given as follows.

$$P_s = v_{sd} i_{sd} = v_{sd} (\frac{\lambda_{sd}}{L_s} - \frac{L_m}{L_s} i_{rd}) \approx -\frac{L_m}{L_s} v_{sd} i_{rd} \tag{0.9}$$

In the above equation, the d-axis flux is nearly equal to zero because the d-axis is aligned along the stator voltage. Also, $\lambda_{sq} \approx -\frac{v_{sd}}{\omega_d}$.

The electromagnetic torque is related to the d-axis rotor current as follows.

$$\tau_{em} = \frac{p}{2} L_m (i_{sq} i_{rd} - i_{sd} i_{rq}) = \frac{p}{2} L_m \left(\left(\frac{\lambda_{sq}}{L_s} - \frac{L_m}{L_s} i_{rq} \right) i_{rd} - \left(\frac{\lambda_{sd}}{L_s} - \frac{L_m}{L_s} i_{rd} \right) i_{rq} \right) \approx -\frac{p}{2} \frac{1}{\omega_d} \frac{L_m}{L_s} v_{sd} i_{rd} \tag{0.10}$$

The reactive power is related to q-axis current as follows.

$$Q_s = -v_{sd} i_{sq} = -v_{sd}\left(\frac{\lambda_{sd}}{L_s} - \frac{L_m}{L_s} i_{rq}\right) \approx \frac{v_{sd}^2}{\omega_d L_s} + \frac{L_m}{L_s} v_{sd} i_{rq} \tag{0.11}$$

Using (1.9) to (1.11), the d-axis and q-axis rotor current references are given as follows.

$$i_{rd}^* \approx -\frac{L_s}{L_m}\frac{P_s^*}{v_{sd}} \approx -\frac{2}{p}\frac{L_s}{L_m}\omega_d \frac{\tau_{em}^*}{v_{sd}} \tag{0.12}$$

$$i_{rq}^* \approx \frac{L_s}{L_m}\frac{1}{v_{sd}}\left(Q_s^* - \frac{v_{sd}^2}{\omega_d L_s}\right) \tag{0.13}$$

The rotor current to voltage transfer functions are given as:

$$G_i(s) = \frac{1}{R_r + s\sigma L_r} \tag{0.14}$$

In this example, it is assumed that the machine is running at the rated slip in generation mode (negative slip) initially, with zero voltage applied at rotor terminals. Also, the stator is assumed to be connected to the grid which supplies rated voltage applied at stator terminals. Assume that the stator voltage vector and rotor A-axis are aligned along the stator a-axis initially. Based on these conditions, the initial d and q axes stator and rotor currents can be determined which are computed in the ‘EX7_3calc.m’ file. Then, the PI controllers for the rotor currents have been computed using the transfer function in equation (1.14).

In the example Simulink model, the d-axis stator current reference and reactive power reference have been given. The q-axis current reference is computed from the reactive power reference, using equation (1.13) in the model. Then, the PI controllers and compensation terms are applied to get the rotor voltage references which are then fed to the ‘DFIG’ subsystem.