Appendix 2-A

Example 2-2

For the "test" machine given in Chapter 1, simulate the induction machine start-up in MATLAB using the equations derived in this chapter from a completely powered down state with no external load connected to it. Plot rotor speed, electromagnetic torque, stator and rotor currents.

Solution:

The parameters of the "test" machine in Chapter 1 are as follows:

Power:	1.5 MW
Voltage:	690 V (L-L, rms)
Frequency:	60 Hz
Phases:	3
Number of Poles:	6
Full-Load Slip	1 %
Moment of Inertia	$70\,kg \cdot m^2$

Per-Phase Circuit Parameters:

$R_s = 0.002\,\Omega$

$R_r = 0.0015\,\Omega$

X_ℓ 5Ω

X_ℓ 17Ω

$X_m = 0.86\,\Omega$

The stator and rotor flux vector $\boldsymbol{\lambda}$ can be written in terms of stator and rotor current vector I and the inductance matrix **L** as

$$\boldsymbol{\lambda} = \mathbf{L}\mathbf{I} \tag{1}$$

The vector $\boldsymbol{\lambda}$ can be written as

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_A \\ \lambda_B \\ \lambda_C \end{bmatrix} \tag{2}$$

where λ_x, [x=a,b,c] denotes the flux of stator phases, while λ_X, [X=A,B,C] denotes the fluxes of rotor phases. Applying the same notations to the currents vector

$$\mathbf{I} = \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_A \\ i_B \\ i_B \end{bmatrix} \tag{3}$$

The inductance matrix $\mathbf{L}$ is given as follows:

$$\mathbf{L} = L_{\text{m,1-phase}} \begin{bmatrix} 1+k_{ls} & \cos(\alpha) & \cos(2\alpha) & \cos(\theta_m) & \cos(\theta_m+\alpha) & \cos(\theta_m+2\alpha) \\ \cos(2\alpha) & 1+k_{ls} & \cos(\alpha) & \cos(\theta_m+2\alpha) & \cos(\theta_m) & \cos(\theta_m+\alpha) \\ \cos(\alpha) & \cos(2\alpha) & 1+k_{ls} & \cos(\theta_m+\alpha) & \cos(\theta_m+2\alpha) & \cos(\theta_m) \\ \cos(\theta_m) & \cos(\theta_m+2\alpha) & \cos(\theta_m+\alpha) & 1+k_{lr} & \cos(\alpha) & \cos(2\alpha) \\ \cos(\theta_m+\alpha) & \cos(\theta_m) & \cos(\theta_m+2\alpha) & \cos(2\alpha) & 1+k_{lr} & \cos(\alpha) \\ \cos(\theta_m+2\alpha) & \cos(\theta_m+\alpha) & \cos(\theta_m) & \cos(\alpha) & \cos(2\alpha) & 1+k_{lr} \end{bmatrix} \tag{4}$$

where

$L_{\text{m,1-phase}}$ = per phase magnetizing inductance of the induction machine,
θ_m = electrical angle between the stator a-axis and rotor A-axis,
k_{ls} = ratio of L_{ls} (stator leakage inductance) and $L_{\text{m,1-phase}}$,
k_{lr} = ratio of L_{lr} (rotor leakage inductance) and $L_{\text{m,1-phase}}$, and
$\alpha = \dfrac{2\pi}{3}$

The voltage current equations can be written in vector form as

$$\mathbf{V} = \frac{d\boldsymbol{\lambda}}{dt} + \mathbf{RI} \tag{5}$$

where

$$\mathbf{V} = \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_A \\ v_B \\ v_C \end{bmatrix} \tag{6}$$

and,

$$\mathbf{R} = \begin{bmatrix} R_s & 0 & 0 & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & 0 & 0 \\ 0 & 0 & R_s & 0 & 0 & 0 \\ 0 & 0 & 0 & R_r & 0 & 0 \\ 0 & 0 & 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & 0 & 0 & R_r \end{bmatrix} \tag{7}$$

In above matrices, v_x ,[x=a,b,c] represents stator per phase voltage and v_X ,[X=A,B,C] represents per phase rotor voltage. Also, R_s and R_r are the per phase stator and rotor resistances, respectively. It should be noted that for a squirrel cage induction machine, the rotor voltages will be zero.

The electromagnetic torque (T_{em}) is as follows:

$$T_{em} = 0.5\mathbf{I}^T \frac{d\mathbf{L}}{d\theta_{\text{mech}}}\mathbf{I} = 0.5\frac{p}{2}\mathbf{I}^T \frac{d\mathbf{L}}{d\theta_m}\mathbf{I} \tag{8}$$

where θ_{mech} is the mechanical angle between the stator a-axis and rotor A-axis and p is the number of poles in the stator. For a 2 pole machine, the mechanical angle θ_{mech} equals the electrical angle θ_m .

Finally, the mechanical equation relating the electromagnetic torque to load torque (T_L) is given below

$$T_{em} = T_L + J\frac{d\omega_{\text{mech}}}{dt} \tag{9}$$

where J is the inertia of the machine and mechanical load and ω_{mech} is the mechanical speed of the rotor. Equations above allow modeling an induction machine.

These equations are used to create a MATLAB function file EX2_2.m to simulate the startup at no-load of an induction machine with parameters given in Table 1. The MATLAB file EX2_2.m uses the function file EX2_2function.m and generates the simulation results for the start up as shown in Fig. 1. Run the file EX2_2.m to get the following results.

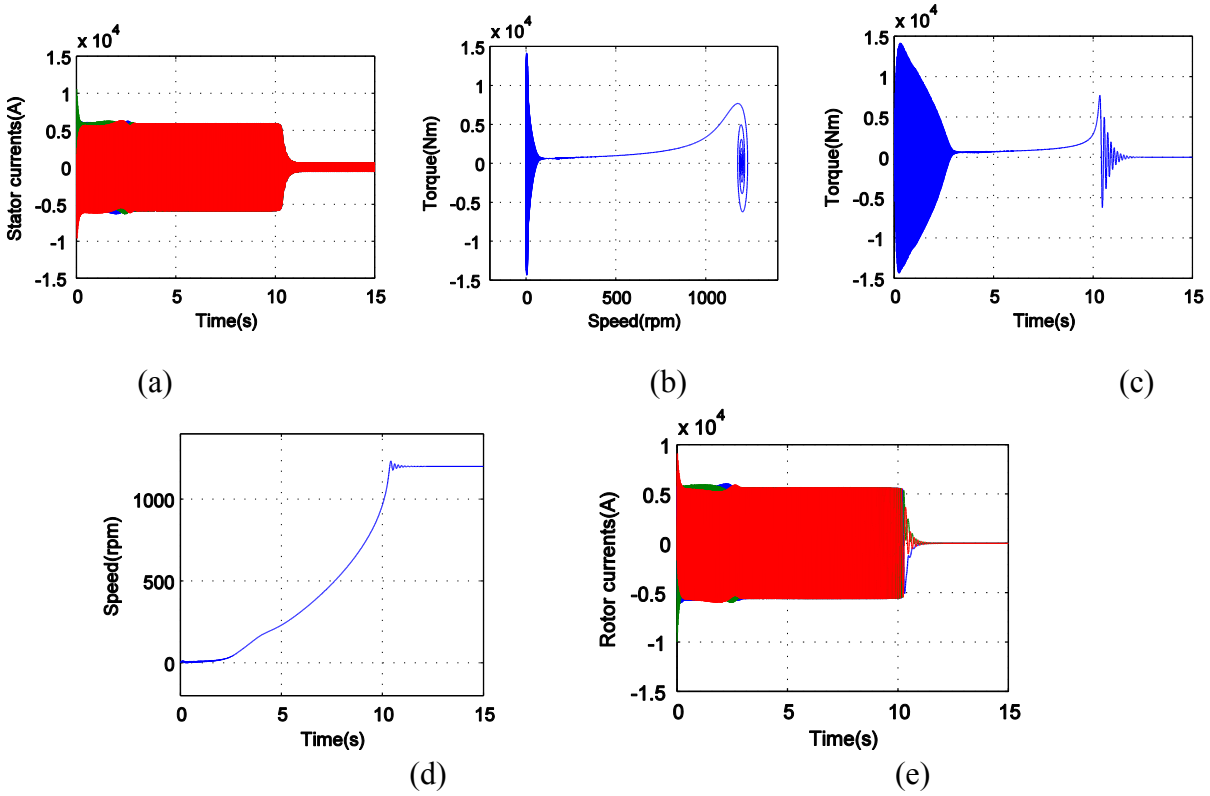

Fig. 1 Simulation results for no-load startup of induction machine (a) Stator currents vs time (b) Rotor speed vs electromagnetic torque (c) Electromagnetic torque vs time (d) Speed vs time (e) Rotor currents vs time